

## Math 109 Review Problems

1. Annual car insurance for a 2009 Honda Civic for an experienced driver in a small city costs  $y = 59x + 685$  (where  $y$  is in dollars and  $x$  is the number of years after 2009.)

- What was the cost in 2016?
- In which year will the cost be \$ 1334?
- Interpret the  $y$ -intercept

2. Graph the feasible set for the following system of inequalities:

$$\begin{aligned}x - 1 &\geq 0 \\y &\geq 2 \\3x + 6y &\leq 18\end{aligned}$$

3. Find the intersection of the lines:

$$\begin{aligned}2x + 3y &= 11 \\-4x + 2y &= 10\end{aligned}$$

4. Demand for a commodity is 5 million units at a price of \$6.25 and 1 million units at a price of \$9.25. Let  $q$  be the quantity in millions and  $p$  be the price in dollars. Express  $p$  in terms of  $q$ .

5. FunPlus makes board games and puzzles. Board games take 4 hours of assembly and 4 hours of testing. Puzzles take 6 hours of assembly and 2 hours of testing. Each day there are 84 hours for assembly and 52 hours for testing available. How many board games and puzzles should be produced each day to maximize the total number of items produced?

6. Let  $U = \{\text{all students}\}$ ,  $B = \{\text{business students}\}$  and  $Y = \{\text{students who are under 20}\}$ . Describe the following set in words:  $B \cap Y'$ .

7. Two thousand first-year students are enrolled in a math or English course. Suppose 800 are taking both math and English while 1200 are taking English. How many are taking a math course?

8. Draw a Venn diagram of the following situation. Of 135 graduating business students:

90 took a Finance course

92 took an Accounting course

75 took a Law course

59 took Finance and Accounting

44 took Finance and Law

45 took Accounting and Law

23 took all three

9. How many 9-digit palindromes are there? Note that the leading digit cannot be zero.

10. How many ways are there to:

a) Choose five of eight books?

b) Choose five of eight books and arrange them in a row on a shelf?

11. Select 6 cards from a standard deck. How many 6-card hands consist of two three-of-a-kind, for example 777KKK?

12. Toss a coin 5 times and observe heads or tails on each toss.

a) How many outcomes are in the sample space?

b) Let E: the first toss is tails. Let F: the last three tosses are heads. Are E and F mutually exclusive? Explain.

13. An experiment has three possible outcomes: A, B and C. Outcome B is four times as likely as outcome A and  $\Pr(C)=0.1$ . Find the probability distribution.

14. A coin is tossed ten times. Find the probability of getting exactly five heads.

15. A manufactured item consists of three independent parts. In a year of use: Part A fails 2% of the time, Part B fails 5% of the time and Part C fails 7% of the time. Find the probability that none of the three parts fail in a year of use.

16. Twenty percent of the emails you receive are spam. The spam-detector is correct 98% of the time, for both spam and non-spam emails. If an email is tagged as spam, what is the probability that it actually is spam?

17. There are three sections of English 999.

Section 1 contains 30% of all English 999 students; 10% of Section 1 students are business students.

Section 2 contains 45% of all English 999 students; 80% of Section 2 students are business students.

Section 3 contains 25% of all English 999 students; 20% of Section 3 students are business students.

Find the probability that a business student is in Section 2 of English 999.

18. You insure a laptop worth \$2,000 against theft for one year by paying a premium of \$160. The probability of theft is 4.9%. Let  $X$  be your net gain on the insurance policy. Find the probability distribution of  $X$  and the expected value of  $X$ .

19. Your company has to deliver 18 projects next week. Each project is independent and has a 95% chance of being completed on time. Find the probability that at least 16 of the 18 projects are completed on time.

20. Solve the following system using Gauss-Jordan Elimination:

$$\begin{aligned}x - 3y + 4z &= 7 \\4x - 10y + 10z &= 28 \\-3x + 9y - 5z &= -42\end{aligned}$$

21. Solve the following system using Gauss-Jordan Elimination:

$$\begin{aligned}4x + 4y &= 0 \\3x + 5y + 4z &= -6 \\2x + y - 2z &= 3\end{aligned}$$

22. Lisa earned 92 on Test 1 and 78 on Test 2. Max earned 62 on Test 1 and 73 on Test 2. Their term marks are calculated by:

Term Mark =  $0.3(\text{Test 1 Mark}) + 0.7(\text{Test 2 Mark})$ . Use a matrix multiplication to calculate the term marks for both students.

23. Find  $A^{-1}$  and use it to solve the system:

$$\begin{aligned}6x + 3y &= 30 \\-3x + 6y &= -45\end{aligned}$$

24. 85% of current TruLight customers will buy their next lightbulb from TruLight and 15% will switch to another brand. Five percent of non-TruLight customers will switch to TruLight when they buy their next lightbulb.

- a) Draw a transition diagram
- b) Find the transition matrix
- c) 40% of lightbulb customers currently use TruLight. What percentage of customers will select TruLight for their next lightbulb purchase?

25. Given the transition matrix  $P = \begin{bmatrix} 0.5 & 0.5 \\ 0.25 & 0.75 \end{bmatrix}$

- a) Find the stationary matrix
- b) As  $k \rightarrow \infty$ , what does  $S_k$  look like?
- c) Find the limiting matrix  $\bar{P}$

26. You buy a 180-day T-Bill with a maturity value of \$6,000 for \$5,820. What is the simple annual interest rate?

27. \$16,000 grows to \$20,000 over 6 years with quarterly compounding. Find the annual nominal interest rate.

28. Beiyan's investment account earns 2% interest, compounded annually. Beiyan makes 17 deposits of \$4500 at the end of each year, then the account earns interest for 28 more years. What is the value of the investment after 45 years?

29. You want your retirement fund to pay \$12,000 quarterly for 25 years. Your retirement fund earns 8% interest, compounded quarterly. How much do you need in your retirement fund on the day of your retirement?

30. Let p: Sooke has an airport.

Let q: Langford has an airport.

Write the following statements symbolically:

- a) Neither Sooke nor Langford have an airport.
- b) It is not the case that Sooke and Langford both have an airport.
- c) If Langford has an airport then Sooke does not.
- d) Sooke has an airport or Langford does not.

31. Construct a truth table for  $(p \vee \sim q) \oplus (q \wedge r)$

32. Let  $p$ : Jo applies for the job.

Let  $q$ : Jo gets for the job.

Write each statement symbolically, using  $p, q, \rightarrow$  and  $\leftrightarrow$ .

- a) Jo gets the job if Jo applies for the job.
- b) Jo gets the job only if Jo applies for the job.
- c) Jo applying is necessary for Jo to get the job.
- d) Jo applying is sufficient for Jo to get the job.
- e) Jo gets the job if and only if Jo applies for the job.

33. Use a truth table to show that  $p \wedge (q \vee r)$  is logically equivalent to  $(p \wedge q) \vee (p \wedge r)$ .

34. Find the equation of the line through the points  $(5, 6.25)$  and  $(1, 9.25)$

35. Draw a Venn diagram for  $B \cup (A \cap C')$

36. A startup company consists of six businesspeople and five coders. How many ways are there to form a team of four employees containing at least two coders?

37. A student has five different business textbooks and four different novels. In how many ways can the books be arranged in a row on a shelf so that all the novels are grouped together?

38. Four companies pick a conference date in January. Find the probability that at least two companies pick the same date.

39. At a three-year college students were surveyed on which year they are in and whether they own a car. The responses were:

|             | Car | No Car |
|-------------|-----|--------|
| First Year  | 8   | 22     |
| Second Year | 13  | 12     |
| Third Year  | 15  | 5      |

- a) Find the probability that a student who owns a car is not in first year.
- b) Are the events **Car** and **Third Year** independent?

40. Find  $A + B$  and  $AB$  (if they exist) given:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 6 & 1 \\ -7 & 5 & -2 \end{bmatrix}$$

41. Find  $A^{-1}$  given:

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 1 & 2 \\ 6 & 3 & 5 \end{bmatrix}$$

42. An account has an interest rate of 7.5%, compounded monthly.

- a) If you deposit \$10,000 today how much will you have in five years?
- b) How much should you deposit today if you want to have \$10,000 in five years?
- c) What monthly deposit should you make in order to have \$10,000 in five years?
- d) What monthly payment would you receive if the account currently has \$10,000 in it and the money is paid out over five years?