

Math 109 Practice Problems

1. Annual tourism revenue on a tropical island is modelled by $y = 1.5x + 2.3$, where y is in millions of dollars and x is the number of years after 2005.

- What was the annual tourism revenue in 2012?
- In which year will the revenue be \$ 26.3 million?
- Interpret the y -intercept

2. Graph the feasible set for the following system of inequalities:

$$\begin{aligned}x &\geq 0 \\y &\geq 0 \\-2x + 2y &\leq 4 \\2x + y &\leq 8\end{aligned}$$

3. Find the coordinates of the vertices for the feasible set above.

4. A salesperson earns \$400 per week plus 5% commission on their sales. Let x denote their weekly sales and y denote weekly earnings (both in dollars).

- Express y in terms of x
- What amount of sales will give the salesperson \$515 in earnings?

5. Sarah has \$45,000 in total to invest in low-, medium- and high-risk assets. The amount in low-risk assets is at most \$5,000 more than the amount in medium-risk assets. The amount in low- and medium-risk assets total at least \$25,000. The amount in medium- and high-risk assets total at most \$35,000. Annual rates of return are 2%, 4% and 8% for the three asset classes. How much should be invested in each asset class to maximize Sarah's annual return?

6. Let $U = \{\text{all people}\}$, $M = \{\text{people who like math}\}$, $S = \{\text{people who like science}\}$ and $B = \{\text{people who like business}\}$. Write the following set in set-theoretic notation: the set of people who like math or business, but not science.

7. Use De Morgan's Laws to simplify $(S' \cup T)'$

8. Draw a Venn diagram for the following situation. Of 200 job applicants:
50 have coding experience
15 have presentation and international experience
11 have international and coding experience
20 have international experience only
10 have presentation and international but not coding experience
9 have presentation and coding but not international experience
30 have no experience.

9. How many 5-letter words (including nonsense words) can be formed from A,B,C,D,E,F,G if:

- a) repetition is allowed?
- b) repetition is not allowed?

10. Simplify the following expressions:

- a) $C(n, n - 3)$
- b) $\frac{P(n,3)}{P(n-1,3)}$

11. Toss a coin 9 times. How many outcomes have at least 3 heads?

12. A company has five employees: A,B,C,D,E. Three are selected for a special project.

- a) Write out the sample space
- b) Let G: B is chosen and C is not. Write out the event G

13. Toss a coin 4 times. Record the number of heads that appear. Determine the probability distribution.

14. Fifteen people are randomly selected. Find the probability that at least two of them share a birthday. Assume that a year has 365 days and disregard the year of birth.

15. A plane has two independent engines. Engine 1 fails on 4% of flights while Engine 2 fails on 5% of flights. Find the probability that at least one engine works properly on the next flight.

16. Ten percent of all crimes in Plainville are violent. The other 90% of crimes are nonviolent. Of the violent crimes, 95% are reported to police. Of the nonviolent crimes only 40% are reported. What is the probability that a crime reported to police is violent?

17. A company has three divisions.

Division A has 20% of all the company's employees; 13% of Division A employees are bilingual.

Division B has 75% of all the company's employees; 22% of Division B employees are bilingual.

Division C has 5% of all the company's employees; 89% of Division C employees are bilingual.

Find the probability that a bilingual employee works for Division C.

18. You insure a used car worth \$8,000 against theft for one year by paying a premium of \$240. The probability of theft is 2.7%. Let X be your net gain from the insurance policy. Find the probability distribution of X and the expected value of X .

19. A basketball player is successful on 62% of his free throws and does not improve with practice. Find the probability that he is successful on at most seven of his next nine attempts.

20. Solve the following system using Gauss-Jordan Elimination:

$$\begin{aligned}3x - 4y + z &= 25 \\2x + 4y + z &= -16 \\x + 5z &= 11\end{aligned}$$

21. Solve the following system using Gauss-Jordan Elimination:

$$\begin{aligned}2x + 2y - 6z &= 18 \\2x + 3y - 12z &= 22\end{aligned}$$

22. A client has 1,000 shares of Abex, 500 shares of Biolinx and 5,000 shares of Campint. Stock prices (in dollars) over three days appear below:

	Tuesday	Wednesday	Thursday
Abex	50	42	48
Biolinx	100	102	97
Campint	4	5	4

Use a matrix multiplication to find the total value (in dollars) of the client's portfolio on the three days.

23. Find A^{-1} and use it to solve the system:

$$\begin{aligned}x + 2y + 2z &= 20 \\x + y + z &= 7 \\2x + 2y + z &= 7\end{aligned}$$

24. If a stock price increases one day, the probability that it increases the next day is 0.75. If a stock price decreases one day, the probability that it decreases the next day is 0.45.

- Draw a transition diagram
- Find the transition matrix
- 30% of stock prices increased today. What percentage will decrease two days from now?

25. Given the transition matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.2 & 0.8 & 0 \end{bmatrix}$

- Find the stationary matrix
- As $k \rightarrow \infty$, what does S_k look like?
- Find the limiting matrix \bar{P}

26. You give Al a \$10,000 90-day loan at 3% simple annual interest. Thirty days later you sell the loan to Bob for \$10,020. What simple annual interest rate will Bob earn?

27. Find the annual percentage yield for:

- 6.25% compounded quarterly
- 4.9% compounded continuously

28. An investor wants to double her money in six years. What annual nominal interest rate is required if interest is compounded quarterly?

29. You make payments of \$3500 into your account at the end of each quarter. Your account earns 3% interest, compounded quarterly. How much is in your account after 20 years? How much of it is interest?

30. A family took out a 25-year \$400,000 mortgage with monthly payments at 3% interest, compounded monthly. How much do they owe after 20 years?

31. Let p : Marketing reports to Finance.

Let q : Sales reports to Finance.

Write the following statements symbolically:

- a) If Sales reports to Finance then Marketing does not.
- b) Neither Sales nor Marketing report to Finance.
- c) It is not the case that Sales and Marketing both report to Finance.
- d) Marketing reports to Finance or Sales does not.

32. Construct a truth table for $(p \vee q) \oplus (\sim p \wedge r)$

33. Statement p is false and statement q is true. State the truth value of the following:

- a) $p \rightarrow q$
- b) $q \rightarrow p$
- c) $(p \oplus q) \rightarrow p$
- d) $(p \oplus q) \rightarrow q$
- e) $(\sim p) \leftrightarrow q$
- f) $p \leftrightarrow (\sim q)$
- g) $p \leftrightarrow q$

34. Decide whether each statement is logically equivalent to $\sim p \vee q$

- a) $p \rightarrow q$
- b) $q \rightarrow p$
- c) $\sim (p \vee \sim q)$
- d) $\sim (p \wedge \sim q)$

35. Let $U = \{a, b, c, d, e, f, g, h, i\}$, $A = \{a, e, i\}$, $B = \{a, b, c, d, e\}$ and $C = \{a, b, d, f\}$. Find $C' \cap (A \cup B)$.

36. Draw a Venn diagram for $(R' \cap S') \cup T$.

37. Find the equation of the line that is parallel to $4x - 5y = 3$ and that passes through $(-8, 4)$.

38. Write down all the inequalities that apply:

A company makes hockey pucks and soccer balls. Each hockey puck takes 5 hours to manufacture and 1 hour to test. Each soccer ball takes 2 hours to manufacture and 2 hours to test. Each day the company has at most 40 manufacturing hours available and at most 20 testing hours available. Let x be the number of hockey pucks made each day and let y be the number of soccer balls made each day.

39. A feasible set has vertices $(0, 0)$, $(5, 0)$, $(0, 4)$ and $(2, 3)$. Find the maximum value of $5x + 6y$ on the feasible set. Also find the point at which the maximum occurs.

40. Solve the following system using Gauss-Jordan Elimination:

$$x + 2y - z = 3$$

$$2x + 4y - 2z = 6$$

$$x - y + 2z = 0$$