8.3 Future Value of an Annuity and Sinking Funds

Annuity: sequence of equal periodic payments

We'll assume: 1) payments @ end of each period
2) payment period = compounding period

$$FV = PMT \cdot \frac{(1+i)^n - 1}{i}$$

FV = future value ($)
PMT = payment ($)
i = interest rate per period
n = total # of periods

Recall 3% interest $r = 0.03$

$i = \frac{r}{m}$ $\quad n = mt$

Ex: We make semi-annual deposits of $1500 for 30 years. Account pays 5%, compounded semi-annually. Value after 30 years? How much is interest?

$m = 2$ $\quad PMT = 1500$ $\quad t = 30$

$r = 0.05$

$i = \frac{r}{m} = 0.025$ $\quad n = mt = 60$

$$FV = PMT \cdot \frac{(1+i)^n - 1}{i}$$

$$= \frac{1500\left((1+0.025)^{60} - 1\right)}{0.025}$$

$$\approx \$203,987.38$$

Interest = FV − Total Payments = $203987.38 - 60(1500) = \$113,987.38$

Sinking Fund: an account established to save a specified amount of money

Ex: A company needs to replace a $600,000 piece of equipment in 7 years. They make quarterly payments into an account paying 9%, compounded quarterly. How much should each payment be?

$FV = 600,000 \quad t = 7 \quad m = 4$

$r = 0.09$

$i = \frac{r}{m} = 0.0225 \quad n = mt = 28$

$$FV = PMT \cdot \frac{(1+i)^n - 1}{i}$$

$$600,000 = PMT \cdot \frac{(\underbrace{1+0.0225}_{1.0225})^{28} - 1}{0.0225}$$

Recall

$9 = x\left(\frac{3}{4}\right)$

$\frac{4}{3} \cdot 9 = x \frac{3}{4} \cdot \frac{4}{3}$

$12 = x$

$$600{,}000 \cdot \frac{0.0225}{(1.0225^{28} - 1)} = PMT$$

$$PMT = \$15{,}615.15$$