# 6.3 Arithmetic Operations on Matrices

**Size** of a matrix : # rows x # columns

Ex: $A = \begin{bmatrix} 3 & 2 \\ -1 & 4 \\ 0 & 1 \end{bmatrix}$

A is $3 \times 2$

Ex: $\begin{bmatrix} -6 & 2x \\ y+1 & 9 \end{bmatrix} = \begin{bmatrix} -6 & 8 \\ 7 & 9 \end{bmatrix}$

Find $x$ and $y$

Corresponding entries are equal

$2x = 8$ $\quad$ $y+1 = 7$

$\boxed{x = 4}$ $\quad$ $\boxed{y = 6}$

**Square** matrix : $\begin{bmatrix} 2 & 1 \\ 3 & 9 \end{bmatrix}$ etc.

**Identity** matrix : $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ etc.

Ex: $A = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$

$3A = \begin{bmatrix} 3 & 9 \\ -6 & 12 \end{bmatrix}$

$A+B = \begin{bmatrix} 7 & 10 \\ 6 & 13 \end{bmatrix}$

$2A - 5B = \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} -30 & -35 \\ -40 & -45 \end{bmatrix}$ ($2A$, $-5B$)

$= \begin{bmatrix} -28 & -29 \\ -44 & -37 \end{bmatrix}$

Ex: $\begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 & 1 \end{bmatrix}$ is undefined because sizes are different.

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Dot product $\begin{bmatrix} 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 5 \end{bmatrix} = 1(1) + 3(5) = 16$

$\begin{bmatrix} 1 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 6 \\ 2 \end{bmatrix} = 1(-1) + 4(6) + 1(2)$

$= 25$

Ex: $A = \begin{bmatrix} 4 & 6 & 1 \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} r_1 \cdot c_1 & r_1 \cdot c_2 \\ r_2 \cdot c_1 & r_2 \cdot c_2 \\ r_3 \cdot c_1 & r_3 \cdot c_2 \end{bmatrix}$$

← rows of A, columns of B

$4(1)+6(1)+1(2)$

$4(0)+6(1)+1(3)$

$$= \begin{bmatrix} 12 & 9 \\ 8 & 11 \\ 4 & 4 \end{bmatrix}$$

Size of AB: 3x2

A (3x3) B (3x2)

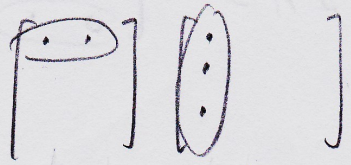
must be equal

size of AB

BA is undefined:

B (3x2) A (3x3)

not equal



Can't multiply

## Notice: $AB \neq BA$ in general

**Ex:** Find CD and DC

$$C = \begin{bmatrix} 1 & 2 & 3 & 6 \\ 1 & 1 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$$

CD is undefined

$$DC = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 6 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 6 \\ 2 & 5 & 9 & 17 \end{bmatrix}$$

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**FACT**

Let $I$ = identity matrix

$AI = A$ and $IA = A$ for any matrix $A$.

**Ex:** $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$$AI = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \checkmark$$

$$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \checkmark$$

# Why Do We Multiply Like This?

Let $A = \begin{bmatrix} 1 & -4 \\ 2 & 3 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$ $B = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

Expand : $AX = B$

$$\begin{bmatrix} 1 & -4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x - 4y \\ 2x + 3y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

$$\begin{cases} x - 4y = 7 \\ 2x + 3y = 6 \end{cases}$$

System of equations

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$AX = B$ is called a <u>matrix equation</u>

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<u>Ex</u>: Write as a matrix equation

$$\begin{cases} 2x - 9y = 12 \\ 7x + 3y = -16 \end{cases}$$

$$AX = B$$

$$\begin{bmatrix} 2 & -9 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -16 \end{bmatrix}$$

Coefficients — Variables (column) — Constants (column)

Ex: An investment firm has 3 clients: X, Y and Z.

Assets (in millions of $):

| | Bonds | Equities | Real Estate |
|---|---|---|---|
| X | 10 | 40 | 50 |
| Y | 30 | 30 | 40 |
| Z | 90 | 5 | 5 |

Last year's return (%):

| | |
|---|---|
| Bonds | 0.02 ← 2% |
| Equities | 0.1 |
| Real Estate | 0.25 |

Calculate total earnings (in millions of $) for each client last year.

$$\begin{matrix} & B & E & RE \\ X \\ Y \\ Z \end{matrix}\begin{bmatrix} \cdot & \cdot & \cdot \\ & & \\ & & \end{bmatrix}\begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}\begin{matrix} B \\ E \\ RE \end{matrix} = \begin{bmatrix} 16.7 \\ 13.6 \\ 3.55 \end{bmatrix}\begin{matrix} X \\ Y \\ Z \end{matrix}$$

Categories must match up