

## 6.2 General Systems of Linear Equations

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Ex: Solve

$$\begin{array}{cccc|c} x & y & z & & \# \\ \hline \textcircled{1} & 2 & 1 & & 9 \\ 1 & 3 & 3 & & 12 \\ 1 & 4 & 5 & & 1 \end{array}$$

$$\begin{array}{l} R_2 - R_1 \\ R_3 - R_1 \end{array} \begin{array}{cccc|c} 1 & 2 & 1 & & 9 \\ 0 & \textcircled{1} & 2 & & 3 \\ 0 & 2 & 4 & & -8 \end{array}$$

$$\begin{array}{l} R_1 - 2R_2 \\ R_3 - 2R_2 \end{array} \begin{array}{cccc|c} & & & & \\ & & & & \\ \hline 0 & 0 & 0 & & -14 \end{array}$$

$$0x + 0y + 0z = -14$$

impossible

The system of equations has  
no solution.

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FACT If you see a row like this, then  
the system has no solution:

$$[0 \ 0 \ | \ \text{nonzero}]$$

$$\text{or } [0 \ 0 \ 0 \ | \ \text{nonzero}]$$

$$\text{or } [\text{all zero} \ | \ \text{nonzero}]$$



Ex: Solve

$$\begin{array}{cccc|c} & x & y & z & \# \\ \textcircled{1} & 2 & 1 & 1 & 9 \\ 1 & 3 & 3 & 3 & 12 \\ 1 & 4 & 5 & 5 & 15 \end{array}$$

$$\begin{array}{l} R_2 - R_1 \\ R_3 - R_1 \end{array} \begin{array}{ccc|c} 1 & 2 & 1 & 9 \\ 0 & \textcircled{1} & 2 & 3 \\ 0 & 2 & 4 & 6 \end{array}$$

$$\begin{array}{l} R_1 - 2R_2 \\ R_3 - 2R_2 \end{array} \begin{array}{ccc|c} 1 & 0 & -3 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}$$

Cannot make this 1  
Elimination is done

Circle the leading 1 in each row

$$\begin{array}{cccc|c} & x & y & z & \# \\ \textcircled{1} & 0 & -3 & -3 & 3 \\ 0 & \textcircled{1} & 2 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

Can solve for  $x$  and  $y$

$z$  is a "free variable"

$$x - 3z = 3$$

$$y + 2z = 3$$

$$\begin{array}{l} z = \text{any value} \\ x = 3 + 3z \\ y = 3 - 2z \end{array}$$

Infinitely many solutions



Ex: Find 3 solutions to the system above

$$(z=0) \quad x=3, y=3, z=0$$
$$(x, y, z) = (3, 3, 0)$$

$$(z=1) \quad (x, y, z) = (6, 1, 1)$$

$$(z=-1) \quad (x, y, z) = (0, 5, -1)$$

Ex: Solve :

a) 
$$\begin{array}{cc|c} x & y & \# \\ 1 & 0 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{array} \leftarrow \text{no info}$$

$$x = 4$$
$$y = 5$$
$$(x, y) = (4, 5)$$

b) 
$$\begin{array}{cc|c} x & y & \# \\ 1 & 0 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{array}$$

no solution



Ex: Solve

$$\begin{array}{ccccc|c} w & x & y & z & \# \\ \hline 2 & 2 & 6 & 14 & 12 \\ 3 & 4 & 15 & 26 & 17 \end{array}$$

$$R_1/2 \quad \begin{array}{cccc|c} \textcircled{1} & 1 & 3 & 7 & 6 \\ 3 & 4 & 15 & 26 & 17 \end{array}$$

$$R_2 - 3R_1 \quad \begin{array}{cccc|c} 1 & 1 & 3 & 7 & 6 \\ 0 & \textcircled{1} & 6 & 5 & -1 \end{array}$$

$$R_1 - R_2 \quad \begin{array}{cccc|c} \textcircled{1} & 0 & -3 & 2 & 7 \\ 0 & \textcircled{1} & 6 & 5 & -1 \end{array}$$

Cricle leading 1's in each row

Can solve for  $w$  and  $x$

$y$  and  $z$  are free variables

$$\begin{aligned} w - 3y + 2z &= 7 \\ x + 6y + 5z &= -1 \end{aligned}$$

$$y = \text{any value}$$

$$z = \text{any value}$$

$$w = 7 + 3y - 2z$$

$$x = -1 - 6y - 5z$$



Ex: Solve

$$\begin{array}{ccc|c} x & y & z & \# \\ \hline 3 & 6 & 6 & 12 \\ 3 & 6 & 8 & 15 \\ 2 & 4 & 4 & 8 \end{array}$$

$$R_1/3 \quad \begin{bmatrix} \textcircled{1} & 2 & 2 & 4 \\ 3 & 6 & 8 & 15 \\ 2 & 4 & 4 & 8 \end{bmatrix}$$

$$\begin{array}{l} R_2 - 3R_1 \\ R_3 - 2R_1 \end{array} \quad \begin{bmatrix} 1 & 2 & 2 & 4 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Can't make this 1  
Move further right

$$R_2/2 \quad \begin{bmatrix} 1 & 2 & 2 & 4 \\ 0 & 0 & \textcircled{1} & 3/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 - 2R_2 \quad \begin{bmatrix} \textcircled{1} & 2 & 0 & 1 \\ 0 & 0 & \textcircled{1} & 3/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Circle leading 1's in each row  
Can solve for  $x$  and  $z$   
 $y$  is a free variable

$$x + 2y = 1$$

|                        |
|------------------------|
| $y = \text{any value}$ |
| $x = 1 - 2y$           |
| $z = 3/2$              |



Ex: A store has 3 gift items priced at \$7, \$10 and \$13. Amanda wants to buy 15 gifts and spend \$150. Give 3 options.

Let  $x =$  # \$7 items  
 $y =$  # \$10 items  
 $z =$  # \$13 items

$$\begin{array}{l} \# : \\ \$ : \end{array} \begin{cases} x + y + z = 15 \\ 7x + 10y + 13z = 150 \end{cases}$$

$$\begin{array}{cccc|c} & x & y & z & \# \\ \textcircled{1} & 1 & 1 & 1 & 15 \\ & 7 & 10 & 13 & 150 \end{array}$$

$$R_2 - 7R_1 \quad \begin{array}{ccc|c} 1 & 1 & 1 & 15 \\ 0 & 3 & 6 & 45 \end{array}$$

$$R_2/3 \quad \begin{array}{ccc|c} 1 & 1 & 1 & 15 \\ 0 & \textcircled{1} & 2 & 15 \end{array}$$

$$R_1 - R_2 \quad \begin{array}{ccc|c} \textcircled{1} & 0 & -1 & 0 \\ 0 & \textcircled{1} & 2 & 15 \end{array}$$

$$\begin{array}{l} x - z = 0 \\ y + 2z = 15 \end{array}$$

$$\begin{array}{l} z = \text{any value} \\ x = z \\ y = 15 - 2z \end{array}$$



Give 3 options (with all variables  $\geq 0$ )

$$1) \quad z=0 \quad x=0 \quad y=15$$

$$2) \quad z=3 \quad x=3 \quad y=9$$

$$3) \quad z=6 \quad x=6 \quad y=3$$

etc.