5.2 Bernoulli Trials and Binomial Distributions

Flashback to Section 3.5

ways to get 2 heads in 4 coin flips?

Choose which flips are heads

$C(4,2) = 6$

HHTT THHT
HTHT THTH
HTTH TTHH

Notation

In this section, text writes $_nC_r$ instead of $C(n,r)$

$\boxed{_4C_2 = 6}$

Bernoulli trial: experiment with only 2 outcomes, usually called "success" and "failure".

Notation

Let p: probability of success

q: " failure

$\boxed{q = 1 - p}$

Ex: Roll a die

Success: roll is 4

Calculate p and q

$p = P(\text{roll is } 4) = \frac{1}{6}$

$q = 1 - p = \frac{5}{6} \longleftarrow P(\text{roll is not } 4)$

Binomial experiment: Sequence of n independent Bernoulli trials

one trial does not affect the others

Ex: Roll a die 7 times

Success: roll is less than 3

Calculate n, p and q

$n = \#\text{ trials} = 7$

$p = P(\text{roll is less than } 3)$

$= \frac{2}{6}$

$= \frac{1}{3}$

$q = 1 - p = \frac{2}{3} \leftarrow P(\text{roll is at least } 3)$

Probability of Exactly x successes in a Binomial Expeiment

$$P(x \text{ successes}) = {}_nC_x \, p^x q^{n-x}$$

$$\begin{cases} n = \#\text{trials} \\ p = P(\text{success on one trial}) \\ q = 1 - p \end{cases}$$

Ex: Roll a die 5 times. Find:

a) P(exactly three 2's)

BINOMIAL
$n = 5$
$p = P(\text{roll a } 2) = \frac{1}{6}$
$q = 1 - p = \frac{5}{6}$
$x = \#\ 2\text{'s rolled}$

$$P(x=3) = {}_nC_x\, p^x q^{n-x}$$
$$= 5C3 \left(\tfrac{1}{6}\right)^3 \left(\tfrac{5}{6}\right)^2$$
$$\approx 0.03$$

b) P(more than three 2's)

$$P(x>3) = P(x=4) + P(x=5)$$
$$= 5C4\left(\tfrac{1}{6}\right)^4\left(\tfrac{5}{6}\right)^1 + 5C5\left(\tfrac{1}{6}\right)^5\left(\tfrac{5}{6}\right)^0$$
$$\approx 0.003$$

c) P(less than four 2's)

$$= P(x<4)$$
$$= P(x=0) + P(x=1) + P(x=2) + P(x=3)$$
$$= 1 - [P(x=4) + P(x=5)]$$
$$\approx 1 - 0.003$$
$$\approx 0.997$$

Explanation

$$P(x \text{ successes}) = {}_nC_x \, p^x \, q^{n-x}$$

Choose which x trials are successes

$P(x \text{ successes})$

$P(n-x \text{ failures})$

Ex: A basketball player makes 65% of his free throws. He takes 3 shots and does not improve with practice.

Let X = # successful free throws.

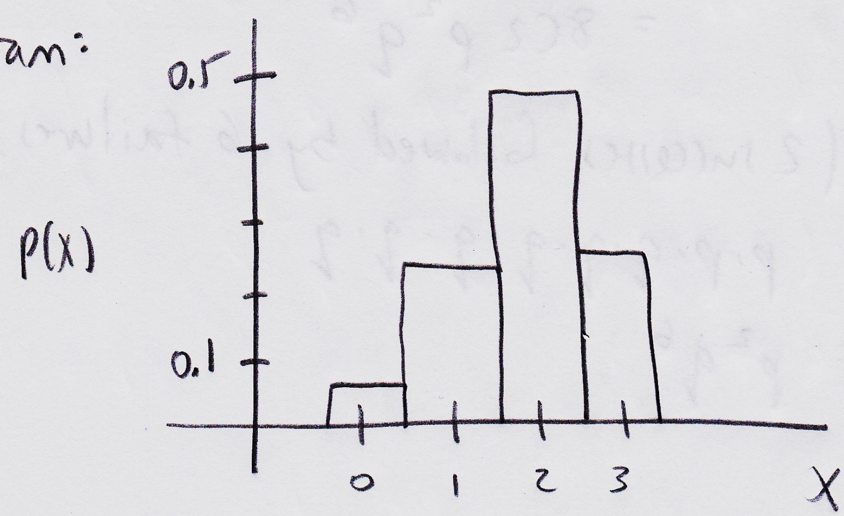
Find the probability distribution of X.

BINOMIAL $n=3$

$p = P(\text{success}) = 0.65 \quad q = 1-p = 0.35$

X	$P(x) = {}_nC_x\, p^x q^{n-x}$
0	${}_3C_0 (0.65)^0 (0.35)^3 = 0.04$
1	${}_3C_1 (0.65)^1 (0.35)^2 \approx 0.24$
2	${}_3C_2 (0.65)^2 (0.35) \approx 0.44$
3	${}_3C_3 (0.65)^3 (0.35)^0 \approx 0.27$

Histogram:



Recall expected value or "mean", written $E(X)$ or μ

$E(X)$ or $\mu = x_1 p_1 + x_2 p_2 + \ldots + x_n p_n$

Represents theoretical average if experiment is repeated infinitely-many times.

FACT

Let X = #successes in a binomial experiment

$E(x)$ or $\mu = np$

Ex: Your company has to deliver 12 projects next week. Each project is independent and has an 80% chance of being completed on time. Find:

a) P(exactly 10 are completed on time)

BINOMIAL $n = 12$

$p = P(\text{on time}) = 0.8$

$q = 1 - p = 0.2$

$x = \#$ on time

$$P(x=10) = {}_nC_x\, p^x q^{n-x}$$

$$= {}_{12}C_{10}\,(0.8)^{10}(0.2)^2$$

$$\approx 0.28$$

b) expected # (or mean #) that will be completed on time

$$E(x) \text{ or } \mu = np \leftarrow$$

only true for binomial experiments

$$= 12(0.8)$$

$$= 9.6$$

Ex: 22% of households in Victoria have pets. Ten households are randomly selected. Find the probability that at least two have pets.

BINOMIAL $n = 10$ $p = 0.22$ $q = 1 - p = 0.78$

x = # households with pets

$$P(x \geq 2) = 1 - P(x=0) - P(x=1)$$
$$= 1 - {}_{10}C_0 (0.22)^0 (0.78)^{10} - {}_{10}C_1 (0.22)^1 (0.78)^9$$
$$\approx 0.68$$