5.1 Expected Value

Notation: $P(X)$ rather than $Pr(X)$ in Chapter 5.

Random variable X: assigns a # to each outcome in the sample space.

Ex: 3 coins are flipped
X = # heads that appear
Find the probability distribution of X

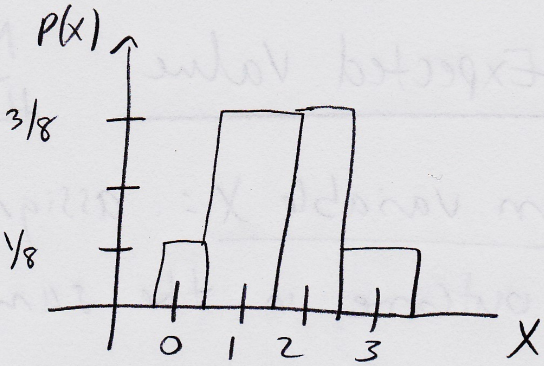
X	Description	# of Outcomes	$P(X)$
0	TTT	1	$1/8 = 0.125$
1	HTT, THT, TTH	3	$3/8 = 0.375$
2	HHT, HTH, THH	3	0.375
3	HHH	1	0.125

total = 8

Probability distribution of X:

X	$P(X)$
0	0.125
1	0.375
2	0.375
3	0.125

Histogram of X:



Expected value

$$E(x) = x_1 p_1 + x_2 p_2 + \dots + x_n p_n$$

theoretical average if the experiment were repeated infinitely-many times

Ex: X = dice roll. Find E(x)

X	P(x)
1	1/6
2	
3	
4	
5	
6	

$$E(x) = 1\left(\tfrac{1}{6}\right) + 2\left(\tfrac{1}{6}\right) + \dots + 6\left(\tfrac{1}{6}\right)$$
$$= 3.5$$

Ex: A box contains 9 \$5 bills and 6 \$10 bills. You pay \$8 and draw a bill.
Let X = your net winnings (\$)
Find $E(X)$

	X	$P(X)$
get \$5	$5-8=-3$	$9/15$
get \$10	$10-8=2$	$6/15$

$$E(X) = -3\left(\frac{9}{15}\right) + 2\left(\frac{6}{15}\right)$$
$$= -1$$

Expect to lose \$1 on average each time you play

Ex: Insure a used car worth \$4,000 against theft for one year by paying a premium of \$112. $Pr(\text{theft}) = 1.3\%$

Let X = your net gain (\$)

Find $E(X)$

	X	$P(X)$
theft	$4000 - 112 = 3888$	0.013
no theft	-112	$0.987 \leftarrow 1 - 0.013$

$$E(X) = 3888(0.013) - 112(0.987)$$
$$= -60$$

Expect to lose \$60 on average each year. ☹

Protected against a large loss. ☺

Ex: A shipment contains 10 good and 5 defective items. Choose 3 items randomly.

Let X = # good items chosen

Find $E(X)$

X	Description	# of Outcomes	P(X)
0	3D	5C3 = 10	10/455
1	2D and 1G	5C2 × 10C1 = 100	100/455
2	1D and 2G	5C1 × 10C2 = 225	225/455
3	3G	10C3 = 120	120/455
		total = 455	

$$E(X) = 0\left(\frac{10}{455}\right) + \dots + 3\left(\frac{120}{455}\right)$$
$$= 2$$

Expect 2 good items, on average.

NOTATION :

We'll write nCr instead of C(n,r) in Chapter 5.

Def

$\boxed{\text{A game is } \underline{\text{fair}} \text{ if the expected net winnings is zero.}}$

<u>Ex</u>: You pay \$1 to roll a die.
If you roll a 1 or 6, you win \$5.
Otherwise, you must pay k more dollars.
Find k so that the game is fair.

X = net winnings	P(x)
4	$\frac{2}{6}$
$-k-1$	$\frac{4}{6}$

$$E(x) = 0$$

$$4\left(\tfrac{2}{6}\right) + (-k-1)\tfrac{4}{6} = 0$$

$$8 + 4(-k-1) = 0$$

$$4(-k-1) = -8$$

$$-k-1 = -2$$

$$-k = -1$$

$$k = 1$$