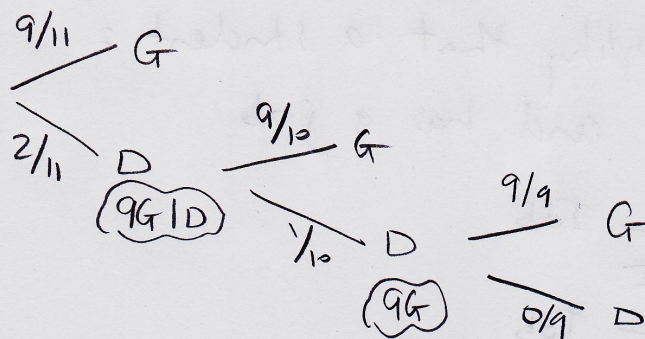


## 4.5 Tree Diagrams

Ex: A shipment contains 9 good and 2 defective items. Items are selected one at a time (without replacement) until a good item is found.

a) Draw a tree diagram.



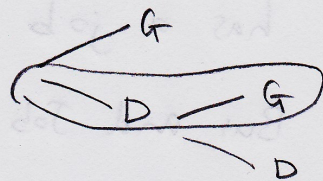
b) Pr(1 item is selected)?

$$\frac{9}{11}$$

c) Pr(2 items are selected)?

Multiply along the path

$$\frac{2}{11} \cdot \frac{9}{10} \approx 0.16$$



d) Pr(3 items are selected)?

$$\frac{2}{11} \cdot \frac{1}{10} \cdot \frac{9}{9} \approx 0.02$$

Ex: At a college

$\frac{3}{5}$  of students are in Business

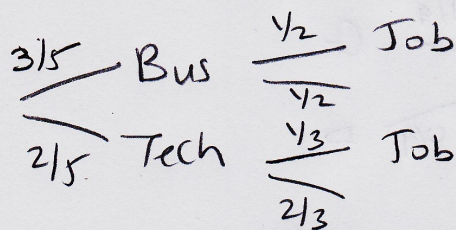
$\frac{2}{5}$  " Technology

$\frac{1}{2}$  of Business students have a job

$\frac{1}{3}$  of Technology "

Find the probability that a student =

a) is in Business and has a job



$$\frac{3}{5} \cdot \frac{1}{2} = 0.3$$

b) has a job

Bus and Job or Tech and Job

$$\frac{3}{5} \cdot \frac{1}{2} + \frac{2}{5} \cdot \frac{1}{3}$$

$$\approx 0.4333$$

and: $\times$
or: $+$

←  
• Multiply along each path  
• Sum all paths to get Pr(job)

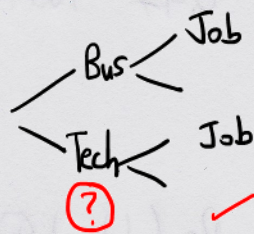
c) is in Business, given that they have a job

$$\Pr(\text{Business} | \text{job}) \\ = \frac{\Pr(\text{Business} \cap \text{job})}{\Pr(\text{job})}$$

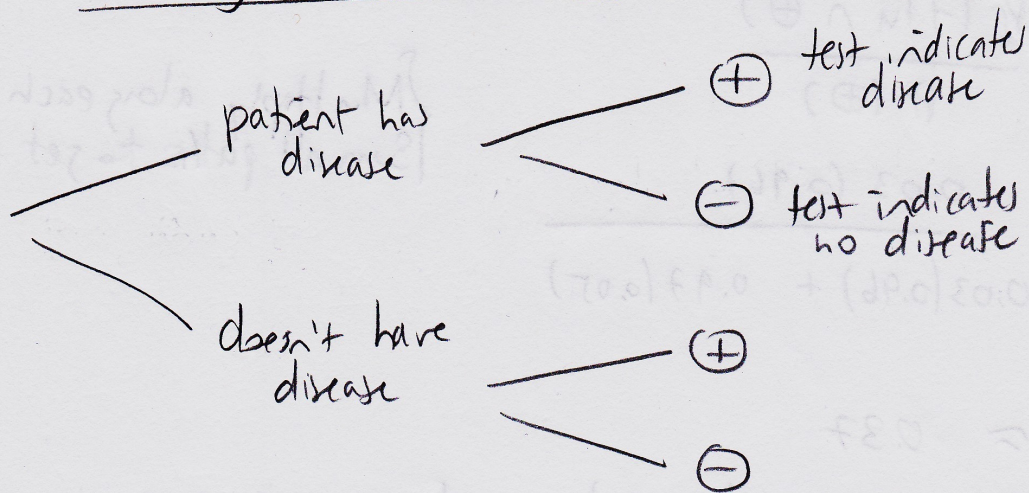
$$\approx \frac{0.3}{0.4333}$$

$$\approx 0.69$$

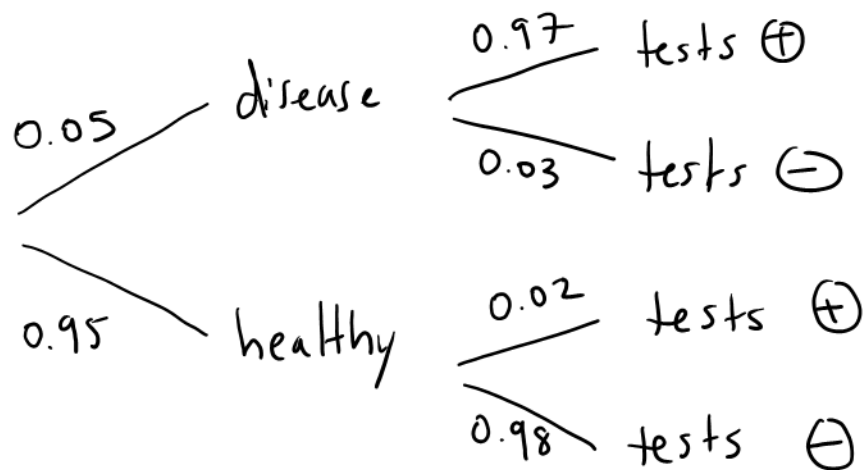
This is a "backwards probability":  
 $\Pr(\text{1st event} | \text{2nd event})$



### Testing for a Disease



Ex: 5% of patients have a certain disease.  
The false-positive rate is 2%.  
The false-negative rate is 3%.  
Draw a tree diagram.



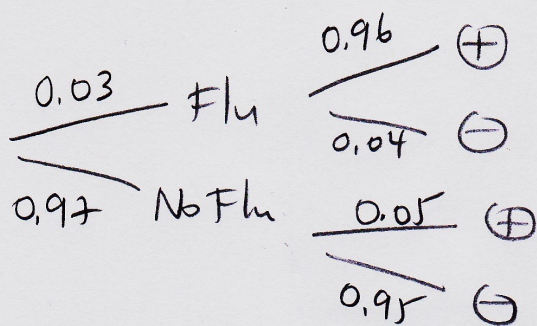
Ex: 3% of students have the flu.

Of people with the flu, 96% test  $\oplus$

no flu, 95% test  $\ominus$

Pr ( person who tests  $\oplus$  actually has the flu ) ?

Want Pr (flu |  $\oplus$ )



$$\begin{aligned} & \text{Pr (flu | } \oplus \text{)} \\ &= \frac{\text{Pr (flu } \cap \oplus \text{)}}{\text{Pr (} \oplus \text{)}} \end{aligned}$$

$$= \frac{0.03 (0.96)}{[0.03(0.96) + 0.97(0.05)]}$$

$$\approx 0.37$$

Surprisingly low. When disease is rare in population, test error becomes significant.

MODIFY: 30% of students have the flu. Now  $\text{Pr (flu | } \oplus \text{)} \approx 0.89$

} Multiply along each path  
} Sum all paths to get Pr( $\oplus$ )