4.3 Calculating Probabilities

When all outcomes are <u>equally likely</u>

$$Pr(E) = \frac{n(E)}{n(S)}$$

$n(E)$ ← # of outcomes in E

$n(S)$ ← # of outcomes in sample space

<u>Ex</u>: A box contains 4 defective and 5 good items. 3 items are randomly selected. Find:

a) Pr(no defective items selected)

9 items in total

$n(S) = C(9,3)$

$n(E)$ = # ways to choose 3 good

$= C(5,3)$

$$Pr(E) = \frac{C(5,3)}{C(9,3)}$$

$$\approx 0.12$$

b) Pr(2 defective items are selected)

$n(E)$ = # ways to choose 2D and 1G

$= C(4,2) \times C(5,1)$

$= 30$

$$Pr(E) = \frac{30}{C(9,3)}$$
$$= 0.36$$

c) Pr(at least 2 defective are selected)

n(E) = # ways to choose 2D and 1G or 3D

$$= C(4,2) \times C(5,1) + C(4,3)$$
$$= 34$$

and: ×
or: +

$$Pr(E) = \frac{34}{C(9,3)}$$
$$= 0.40$$

Ex: A die is rolled 7 times.
Pr(exactly five 3's)?

$n(S) = 6 \times 6 \times \ldots \times 6 = 6^7$

(# options for Roll 1) (Roll 7)

n(E) = # ways to get five 3's

$= C(7,5) \times 5 \times 5 = 525$

(choose which rolls are 3's) (# choices for first non-3) (second non-3)

$$Pr(E) = \frac{525}{6^7}$$

$$\approx 0.002$$

Ex: A 5-card poker hand is dealt. $Pr(2 \text{ pairs})$?

e.g. 88QQK ← cannot be 8 or Q

$n(S)$ = # ways to choose 5 cards

$= C(52,5)$

$n(E) = C(13,2) \times C(4,2) \times C(4,2) \times 44$

- $C(13,2)$: choose denomination for the 2 pairs
- $C(4,2)$: suits for first pair
- $C(4,2)$: suits for second pair
- 44: $52-4-4 = 44$ choices for last card

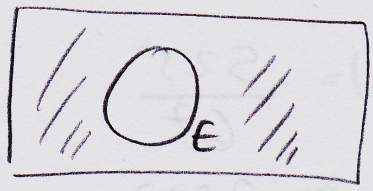
$= 123,552$

$$Pr(E) = \frac{123,552}{C(52,5)}$$

$$\approx 0.058$$

Complement Rule

$Pr(E) = 1 - Pr(E')$



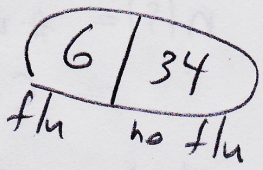
E'

Ex: In a class of 40 students, 6 have the flu. 5 students are randomly selected.

Pr(at least 1 has the flu)?

E: at least 1 has the flu

E': none have the flu



$$Pr(E') = \frac{C(34,5)}{C(40,5)} = 0.42$$

$$Pr(E) = 1 - Pr(E')$$
$$= 0.58$$

Ex: A group of 20 people are randomly selected. Pr(at least 2 share a birthday)?

Notes: Assume 365 days in a year
Disregard year

E: at least 2 share a birthday

E': all 20 birthdays are different

$n(S) = 365 \times 365 \times \dots \times 365 = 365^{20}$

(Person 1) (Person 20)

$n(E') = 365 \times 364 \times \dots$ (20 factors)

$= P(365, 20)$ (Person 1)

$$Pr(E') = \frac{P(365, 20)}{365^{20}} \approx 0.59$$

$$Pr(E) = 1 - Pr(E')$$

$$\approx 0.41$$