

4.2 Assignment of Probabilities

Probability: measure of likelihood

$Pr(\text{event})=0$ means event cannot happen

$Pr(\text{event})=1$ " is guaranteed to happen

Probability distribution:

<u>Outcome</u>	<u>Probability</u>
\vdots	\vdots

Properties of a probability distribution:

- 1) Each probability is between 0 and 1 (inclusive)
- 2) The sum of all probabilities is 1

Ex: Probability Distribution for roll of a die

Outcome	Probability
1	$\frac{1}{6}$
2	$\frac{1}{6}$
3	
4	
5	
6	$\frac{1}{6}$

Ex: Students are polled on their program. Determine the P.D.

Program	#. of Students
Business	13
Technology	18
Nursing	9

↑
Total = 40

Outcome	Probability
Business	$\frac{13}{40} = 0.325$
Technology	$\frac{18}{40} = 0.45$
Nursing	$\frac{9}{40} = 0.225$

Ex: Toss a Coin 3 times. Record # of heads.
Determine the P.D.

Outcome	Description	# of Ways	Probability
0	TTT	1	$\frac{1}{8} = 0.125$
1	H TT, T HT, T TH	3	0.375
2	H HT, H TH, TH H	3	0.375
3	HHH	1	0.125

Total = 8

Outcome	Probability
0	0.125
1	0.375
2	0.375
3	0.125

Ex: Roll a red die and a blue die.

a) Describe the P.D.

red \nearrow $6 \times 6 = 36$ possible outcomes
blue \uparrow

Outcome	Probability
(1,1)	$\frac{1}{36}$
(1,2)	
\vdots	
red \rightarrow (6,6) blue	

b) $\Pr(\text{roll on red is even and roll on blue is greater than roll on red})?$

Call this E

$$E = \{(2, 3), (2, 4), (2, 5), (2, 6), (4, 5), (4, 6)\}$$

$$\begin{aligned}\Pr(E) &= \frac{1}{36} + \frac{1}{36} + \dots + \frac{1}{36} \quad (6 \text{ times}) \\ &= \frac{6}{36} \\ &= \frac{1}{6}\end{aligned}$$

Ex: An unfair four-sided die has the following P.D.

Outcome	Probability
1	0.1
2	0.35
3	0.3
4	0.25

Find:

$$\begin{aligned}\text{a) } \Pr(\text{roll is less than 3}) &= \Pr(\text{roll is 1}) + \Pr(\text{roll is 2}) \\ &= 0.45\end{aligned}$$

$$\begin{aligned}\text{b) } \Pr(\text{roll is odd}) &= \Pr(\text{roll is 1}) + \Pr(\text{roll is 3}) \\ &= 0.4\end{aligned}$$

c) $\Pr(\text{roll is less than 2 and even})$
 $= 0$ ← impossible

Ex: An experiment has possible outcomes A, B and C.

$$\Pr(A) = 0.4$$

B is three times as likely as C.

Find the P.D.

Let $\Pr(C) = x$

$$\Pr(B) = 3x$$

$$\Pr(A) + \Pr(B) + \Pr(C) = 1$$

$$0.4 + 3x + x = 1$$

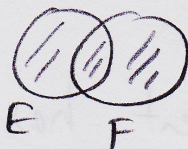
$$4x = 0.6$$

$$x = 0.15$$

Outcome	Probability
A	0.4
B	0.45 ← $3x$
C	0.15 ← x

Inclusion - Exclusion Principle

$$\Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(E \cap F)$$



Ex: $\Pr(E) = 0.6$ $\Pr(F) = 0.5$

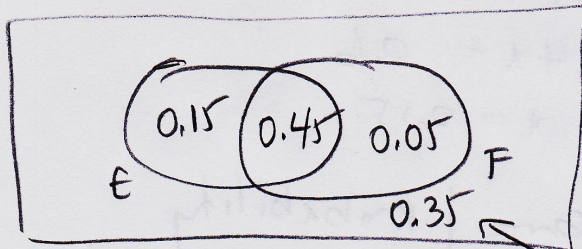
$\Pr(E \cup F) = 0.65$ Find:

a) $\Pr(E \cap F)$

$$\Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(E \cap F)$$

$$0.65 = 0.6 + 0.5 - \Pr(E \cap F)$$

$$\Pr(E \cap F) = 0.45$$



b) $\Pr(E' \cap F)$

$$= 0.05$$

$$1 - 0.15 - 0.45 - 0.05$$

Ex: A company has 2 suppliers.

$$\Pr(\text{Supplier 1 is late}) = 0.2$$

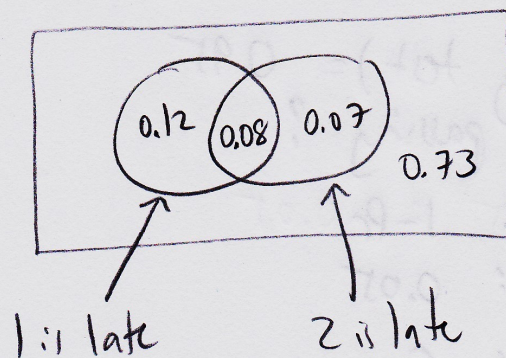
$$\Pr(\text{Supplier 2 is late}) = 0.15$$

$$\Pr(\text{both suppliers are late}) = 0.08$$

Find $\Pr(\text{neither is late})$.

$$\begin{aligned}\Pr(1 \text{ or } 2 \text{ is late}) &= \Pr(1 \text{ is late}) + \Pr(2 \text{ is late}) \\ &\quad - \Pr(\text{both are late}) \\ &= 0.2 + 0.15 - 0.08 \\ &= 0.27\end{aligned}$$

$$\begin{aligned}\Pr(\text{neither is late}) &= 1 - \Pr(1 \text{ or } 2 \text{ is late}) \\ &= 0.73\end{aligned}$$



Odds

"Odds of winning are 2:1" means
You are twice as likely to win as to lose

WIN	WIN	LOSE
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$\underbrace{\hspace{10em}}_{2/3} \qquad \qquad \qquad \underbrace{\hspace{2em}}_{1/3}$

If the odds of an event are $a:b$
then $\Pr(\text{event}) = \frac{a}{a+b}$

Ex: a) Odds of getting a promotion are 2:7
 $\Pr(\text{promotion})?$

$$\frac{2}{2+7} = \frac{2}{9}$$

b) $\Pr(\text{passing test}) = 0.95$
Odds of passing?

$$\Pr : 1 - \Pr$$

$$0.95 : 0.05$$

$$95 : 5$$

($\times 100$)

($\div 5$)

$$19 : 1$$

$$c) Pr(\text{graduating}) = \frac{33}{36}$$

Odds of graduating ?

$$Pr : 1 - Pr$$

$$\frac{33}{36} : \frac{3}{36}$$

$$(\times 36) \quad 33 : 3$$

$$(\div 3) \quad 11 : 1$$