4.1 Experiments, Outcomes, Sample Spaces and Events

The sample space is the set of possible outcomes of an experiment. An event is a subset of the sample space.

Ex: Roll a die. ← "experiment"

Sample space: $S = \{1, 2, 3, 4, 5, 6\}$

Some events:

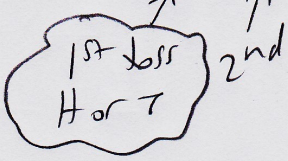
E: roll an even number $E = \{2, 4, 6\}$

F: roll is less than 3 $F = \{1, 2\}$

Ex: Toss a coin 3 times

a) Write out the sample space

possible outcomes = $2 \times 2 \times 2 = 8$



$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

b) Write out E: at most one tail

$E = \{HHH, HHT, HTH, THH\}$

Ex: Four employees A, B, C, D
Choose 2 for a project.

a) Write out the sample space

$S = \{ \{A,B\}, \{A,C\}, \{A,D\}, \{B,C\}, \{B,D\}, \{C,D\} \}$

b) Write out E: A is not chosen

$E = \{ \{B,C\}, \{B,D\}, \{C,D\} \}$

Ex: Suppose an experiment has sample space $S = \{1, 2\}$.
Write out all possible events.

↑ Subsets of S

$\{1,2\}$
$\{1\}$
$\{2\}$
$\emptyset$

Ex: Roll a red die and a blue die.

a) How many outcomes in the sample space?

$6 \times 6 = 36$

(# options for red die) (blue)

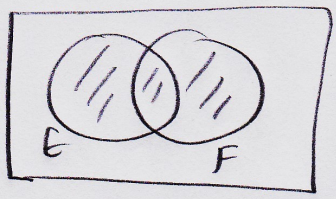
Red \ Blue	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

b) E: rolls sum to 6
Write out E

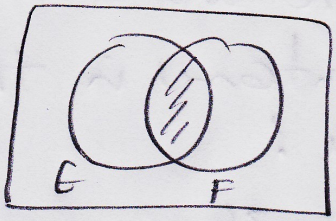
$E = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

(red, blue)

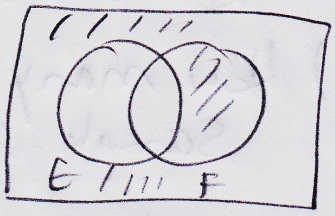
Recall:



$E \cup F$



$E \cap F$



E'

Ex: Cont'd

F: roll on red die is 4

G: roll on red is 3 or 4 and roll on blue is at least 5

a) write out F and G

$F = \{(4,1), (4,2), \ldots, (4,6)\}$

$G = \{(3,5), (3,6), (4,5), (4,6)\}$

(3,5): red ↗, blue ↑

b) Find $F \cup G$

$F \cup G = \{(4,1), (4,2), \ldots, (4,6), (3,5), (3,6)\}$

c) Find $F' \cap G$

F': roll on red die is not 4

$F' \cap G = \{(3,5), (3,6)\}$

Two events E and F are mutually exclusive if $E \cap F = \emptyset$

(no outcomes in common)

Ex: Flip a coin 3 times
Are the events mutually exclusive?

a) E: no heads
F: no tails

$E = \{TTT\}$ $F = \{HHH\}$

$E \cap F = \emptyset$ | Yes, m.e.

f) G: at most one tail
F: no tails

$G = \{HHH, THH, HTH, HHT\}$

$F = \{HHH\}$

$E \cap F \neq \emptyset$ | No, not m.e.