# 3.5 Permutations and Combinations

**Permutation**: Ordered selection of r objects from a group of n objects

# of permutations is written $P(n,r)$

$$P(n,r) = n \times (n-1) \times (n-2) \times \cdots \times (n-r+1)$$

← r factors

**Ex:** $P(40,3) = 40 \times 39 \times 38 = 59,280$

$P(4,4) = 4 \times 3 \times 2 \times 1 = 24$

$P(8,1) = 8$

**Ex:** How many 2 letter "words" can be formed from A, B, C, D if repetition is not allowed?

$P(4,2) = 4 \times 3 = 12$

AB AC AD BC BD CD
BA CA DA CB DB DC

## Factorials

$r! = r \times (r-1) \times \dots \times 2 \times 1$

$3! = 3 \times 2 \times 1 = 6$

$2! = 2 \times 1 = 2$

$1! = 1$

$0! = 1$ by definition

**Combination:** Unordered selection of $r$ objects from a group of $n$ objects

# of combinations is written $C(n,r)$

$$C(n,r) = \frac{P(n,r)}{r!}$$

**Ex:** $C(40,3) = \frac{P(40,3)}{3!} = \frac{40 \times 39 \times 38}{3 \times 2 \times 1} = 9880$

$$C(4,4) = \frac{P(4,4)}{4!} = \frac{4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1} = 1$$

$$C(8,1) = \frac{P(8,1)}{1!} = \frac{8}{1} = 8$$

$$C(n,n-2) = \frac{P(n,n-2)}{(n-2)!} = \frac{n \times (n-1) \times (n-2) \times \dots \times 3}{(n-2) \times (n-1) \times \dots \times 3 \times 2 \times 1}$$

$$= \frac{n(n-1)}{2}$$

Ex: How many ways to choose 2 people from a group of 4?

$$C(4,2) = \frac{P(4,2)}{2!} = \frac{4 \times 3}{2} = 6$$

{A,B} {A,C} {A,D} {B,C} {B,D} {C,D}
= {B,A}

Ex: Interview 20 candidates. How many ways to rank them 1st, 2nd and 3rd?

ordered $P(20,3) = 20 \times 19 \times 18 = 6840$

Ex: Class has 45 students. How many ways to form a 4-person team?

unordered $C(45,4) = \frac{P(45,4)}{4!} = \frac{45 \times 44 \times 43 \times 42}{4 \times 3 \times 2 \times 1} = 148,995$

Ex: On Calculator

a) 8!

[8] [2nd F] [n!] [=] 40,320

b) P(8,2)

[8] [2nd F] [nPr] [=] 56

c) C(8,2)

[8] [2nd F] [nCr] [=] 28

Ex: In a batch of 150 phones, 4 are defective.

a) How many ways to choose 3 phones from the batch?

$$C(150,3) = \frac{P(150,3)}{3!} = \frac{150 \times 149 \times 148}{3 \times 2 \times 1} = 551,300$$

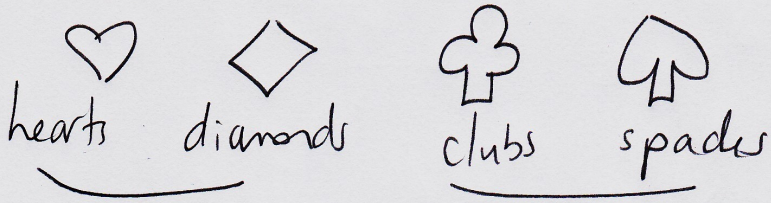
b) How many of these 3 selections contain only defective phones?

→ Choose 3 defective phones

$$C(4,3) = \frac{4 \times 3 \times 2}{3 \times 2 \times 1} = 4$$

## Standard Deck

52 cards divided into 4 suits:



hearts, diamonds, clubs, spades

"red" "black"

Each suit has 13 cards: $A, 2, 3, \ldots, 10, J, Q, K$

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**Ex:** How many 5-card hands have:

a) only hearts

Choose 5 of 13 hearts

$$C(13,5) = \frac{13 \times 12 \times 11 \times 10 \times 9}{5 \times 4 \times 3 \times 2 \times 1} = 1287$$

b) no hearts

Choose 5 of 39 non-hearts

(39 = 52 − 13)

$$C(39,5) = \frac{39 \times 38 \times 37 \times 36 \times 35}{5 \times 4 \times 3 \times 2 \times 1} = 575,757$$

Ex: How many ways to arrange 4 books in a row?

$4 \times 3 \times 2 \times 1 = 4! = 24$

(1st book, 2nd book)

Ex: How many ways to choose 4 of 7 books and arrange them in a row?

Method I: $P(7,4) = 7 \times 6 \times 5 \times 4 = 840$

Method II: $C(7,4) \times 4! = \frac{7 \times 6 \times 5 \times 4}{4!} \times 4! = 840$

($C(7,4)$: choose 4 books; $4!$: arrange them in order)

Ex: 5 students are giving presentations. How many orders are possible if Al goes last and Bob goes first or second?

$\_ \_ \_ \_ \underline{Al}$

(Bob)

2 choices for Bob
Then $3 \times 2 \times 1$ ways to arrange the other 3

Total $= 2 \times 3! = 12$

Ex: How many ways to arrange 4 pairs of people in a row so that each pair is adjacent.

Call people $\underbrace{A_1, A_2}_{\text{pair}}, \underbrace{B_1, B_2}, \underbrace{C_1, C_2}, \underbrace{D_1, D_2}$

e.g. $\underline{B_2 B_1}\ \underline{D_1 D_2}\ \underline{A_1 A_2}\ \underline{C_2 C_1}$

Arrange pairs in $4! = 24$ ways

e.g. BDAC or CABD etc.

Then 2 choices for each pair: $B_1 B_2$ or $B_2 B_1$

$$\text{Total} = 4! \times 2 \times 2 \times 2 \times 2 = 384$$

↑ arrange pairs; ↑ pair A; ↑ B; ↑ C; ↑ D