3.2 A Fundamental Principle of Counting

$n(S)$: # of elements in set S

Ex: $A = \{a, b, c\}$

$n(A) = 3$

$B = \emptyset$

$n(B) = 0$

Inclusion-Exclusion Principle

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Ex: Confirm for $A = \{w, x, y\}$, $B = \{x, y, z\}$

$A \cup B = \{w, x, y, z\}$

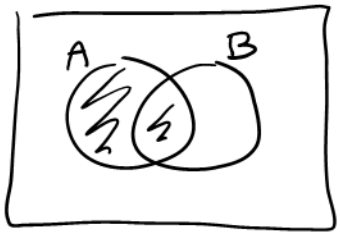
$A \cap B = \{x, y\}$

$n(A \cup B) = 4$

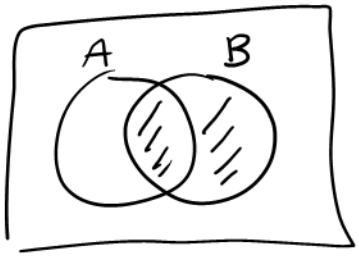
$$n(A) + n(B) - n(A \cap B)$$
$$= 3 + 3 - 2$$
$$= 4$$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ ✓

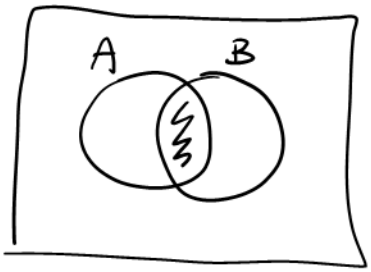
Venn diagram: A way to visualize sets



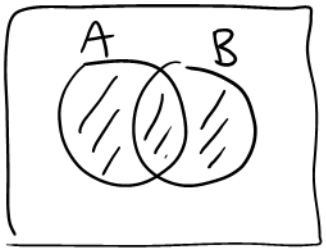
A



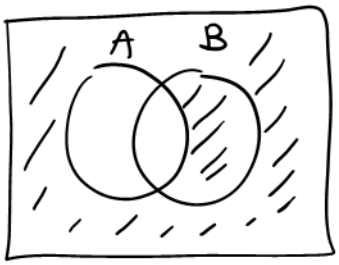
B



$A \cap B$



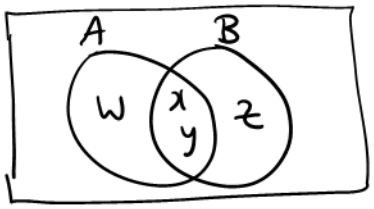
$A \cup B$



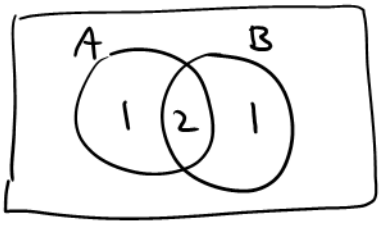
A'

From previous example:

Elements:



Number of elements:



$n(A \cup B) = n(A) + n(B) - n(A \cup B)$

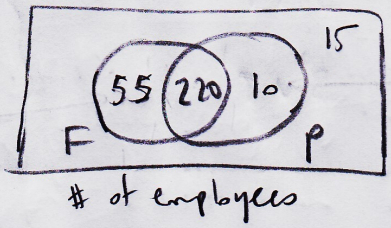
$4 = 3 + 3 - 2$ ✓

Ex: A company has 300 employees; 275 are full-time and 230 are permanent while 285 are full-time or permanent. How many are full-time and permanent?

F = full-time
P = permanent

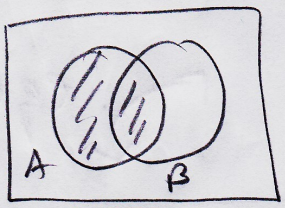
$$n(F \cup P) = n(F) + n(P) - n(F \cap P)$$
$$285 = 275 + 230 - n(F \cap P)$$
$$n(F \cap P) = 220$$

Visually:

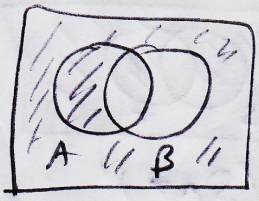


# of employees

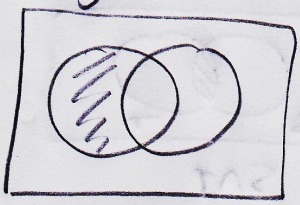
Ex: Draw $A \cap B'$ in a Venn diagram



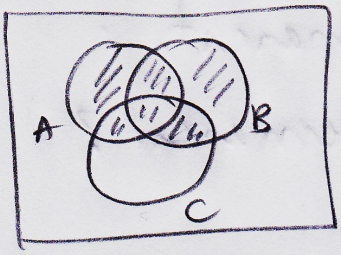
A



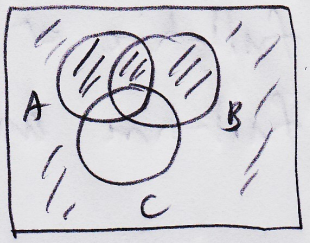
B'


$A \cap B'$

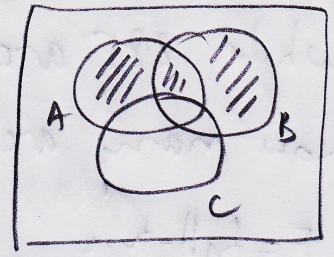
Ex: Draw $(A \cup B) \cap C'$ in a Venn diagram



$A \cup B$



C'



$(A \cup B) \cap C'$

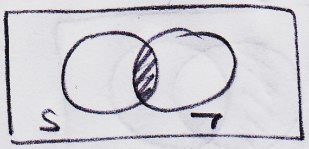
De Morgan's Laws

1) $(S \cup T)' = S' \cap T'$

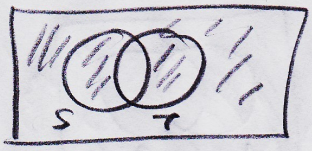
2) $(S \cap T)' = S' \cup T'$

Caution: operation reverse!

Ex: Confirm 2) using Venn diagrams

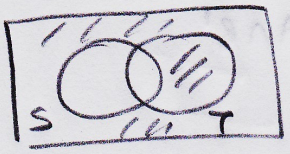


$S \cap T$



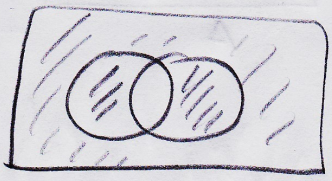
$(S \cap T)'$

equal!



S'

T'



$S' \cup T'$

Ex: Simplify $(A \cup B')'$

$= A' \cap B''$

$= A' \cap B$