2.1 A Linear Programming Problem and

2.2 Fundamental Theorem

Goal: Maximize or minimize a quantity with some restrictions

e.g. maximize profit with limited raw materials

Ex: Each day a company has 60 kg of wood and 100 kg of metal available.

A chair uses 2 kg of wood, 4 kg of metal and yields a \$14 profit.

A table uses 3 kg of wood, 4 kg of metal and yields a \$20 profit.

How many chairs and tables maximize the daily profit?

1) Variables

Let x = # chairs produced each day

y = # tables "

2) Chart

	(x) Chair	(y) Table	Available
Wood (kg)	2	3	60
Metal (kg)	4	4	100
Profit ($)	14	20	

3) Inequalities

max available

wood : $$\underbrace{2x}_{\text{kg for all chairs}} + \underbrace{3y}_{\text{kg for all tables}} \le 60$$

kg/chair

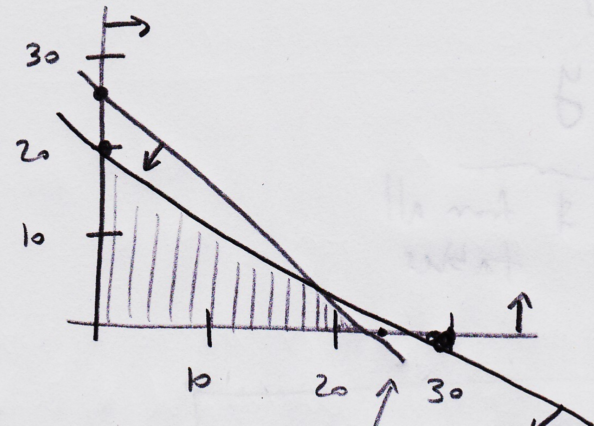
#chairs

metal: $4x + 4y \le 100$

non-negative # chairs, tables : $x \geq 0$, $y \geq 0$

4) Graph Feasible Set

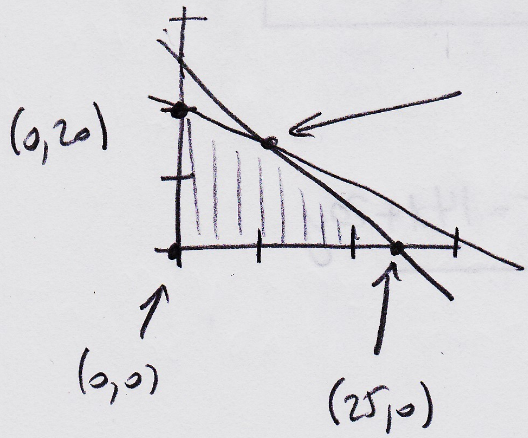
$$2x+3y \le 60$$
$$4x+4y \le 100$$
$$x \ge 0$$
$$y \ge 0$$



$4x+4y=100$
(0, 25)
(25, 0)

$2x+3y = 60$
(0, 20)
(30, 0)

5) Find all vertices



$4x+4y=100$

$y=25-x$

$2x+3y=60$

$y=20-\frac{2}{3}x$

$y=y$

$25-x = 20-\frac{2}{3}x$

Multiply by 3:

$75-3x = 60-2x$

$15 = x$

→ either

$y=10$

(15, 10)

6) Objective Function

Maximize Profit

$$\underbrace{\underset{\$/\text{chair}}{14}\,\underset{\#\text{ chairs}}{x}}_{\$\text{ from all chairs}} + \underbrace{20y}_{\$\text{ from all tables}}$$

Fundamental Theorem

The maximum (or minimum) value of the objective function occurs at one of the vertices of the feasible set

7) Table

Vertices	Profit = $14x + 20y$
(0,0)	0
(0, 20)	400
(25, 0)	350
(15, 10)	410 ← max = 410 @ $x = 15$ $y = 10$

8) Answer

Max profit is \$410 per day from 15 chairs and 10 tables

Ex: Astronauts have two foods: A and B. Food A has 40g protein, 12g fat, 50g carbs per serving and weighs 0.4 kg per serving. Food B: 10g, 15g, 20g, 0.3 kg respectively. Astronauts require at least 120g protein, 60g fat and 200g carbs per day. How many servings of A and B will minimize total mass per day?

1) Variables

x = # servings of A per day

y = " B "

2) Chart

	A (x)	B (y)	Required
(g) Protein	40	10	120
(g) Fat	12	15	60
(g) Carbs	50	20	200
(kg) Mass	0.4	0.3	~~XXXXXX~~

3) Inequalities

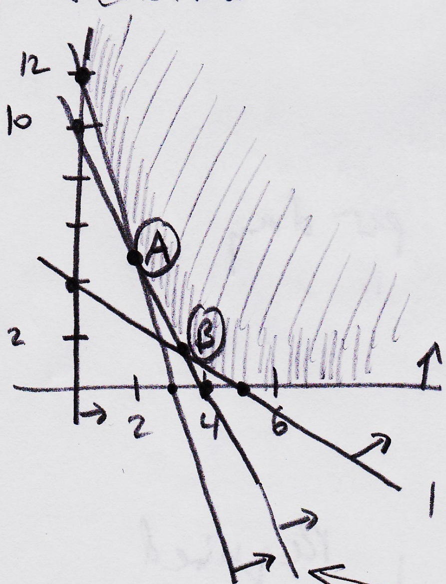
$40x + 10y \geq 120$ ← minimum required

$12x + 15y \geq 60$

$50x + 20y \geq 200$

non-negative: $x \geq 0$, $y \geq 0$

4) Feasible set



$12x + 15y = 60$
(5,0)
(0,4)

$40x + 10y = 120$
(3,0)
(0,12)

$50x + 20y = 200$
(4,0)
(0,10)

5) Vertices

(0,12)

(5,0)

A: $40x + 10y = 120$ $\quad$ $50x + 20y = 200$

$y = 12 - 4x$ $\quad$ $y = 10 - 2.5x$

$y = y$

$12 - 4x = 10 - 2.5x$

Multiply by 2:

$24 - 8x = 20 - 5x$

$4 = 3x$

$\frac{4}{3} = x$

→ either

$y = 12 - 4\left(\frac{4}{3}\right)$

$= \frac{36}{3} - \frac{16}{3}$

$= \frac{20}{3}$

$A = \left(\frac{4}{3}, \frac{20}{3}\right)$

B: $50x + 20y = 200$ $\quad$ $12x + 15y = 60$

$y = 10 - 2.5x$ $\quad$ $y = 4 - \frac{12}{15}x$

$y = 4 - \frac{4}{5}x$

→

$$y = y$$
$$10 - 2.5x = 4 - \frac{4}{5}x$$
Multiply by 10:
$$100 - 25x = 40 - 8x$$
$$60 = 17x$$
$$x = \frac{60}{17}$$

→ either
$$y = 10 - 2.5\left(\frac{60}{17}\right)$$
$$y = \frac{20}{17}$$

$$B = \left(\frac{60}{17}, \frac{20}{17}\right)$$

6) Objective

Minimize $\text{Mass} = 0.4x + 0.3y$

7) Table

Vertices	$\text{Mass} = 0.4x + 0.3y$
$(0, 12)$	3.6
$(5, 0)$	2
$\left(\frac{4}{3}, \frac{20}{3}\right)$	$\frac{76}{30} \approx 2.53$
$\left(\frac{60}{17}, \frac{20}{17}\right)$	$\frac{30}{17} \approx 1.76$ ← minimum @ $x = \frac{60}{17}$ $y = \frac{20}{17}$

8) Answer

$\frac{60}{17}$ servings of A and $\frac{20}{17}$ servings of B per day
(Minimum mass is $\frac{30}{17}$ kg)