4.3 Calculating Probabilities

#17 3 people are chosen. What is the probability that at least 2 of them are born on the same day of the week?

ORDER MATTERS

$n(S) = 7 \times 7 \times 7 = 343$

E = at least 2 born on same day

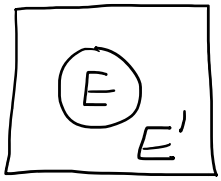
E' = all born on different days

$n(E') = 7 \times 6 \times 5 = 210$

Person 1 Person 2 Person 3

$$Pr(E') = \frac{n(E')}{n(S)} = \frac{210}{343} \approx 0.61$$

★ $Pr(E) = 1 - Pr(E')$ Complement Rule



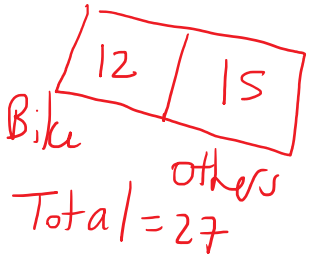
$$Pr(E) = 1 - 0.61$$
$$= 0.39 \quad \text{or } 39\%$$

or .3878

Ex: At a company, 12 employees bike to work
6 " transit "
9 " drive "

Three employees are randomly selected.
Probability that at least 2 bike to work?

UNORDERED



$n(S) = C(27, 3) = 2925$
Select 3 employees

$n(E) = C(12,2) \times C(15,1) + C(12,3)$
2 Bike and 1 other OR 3 Bike

$= 66 \times 15 + 220$

$= 1210$

$Pr(E) = \frac{n(E)}{n(S)}$ ← # desirable outcomes / ← total # of outcomes

$$= \frac{1210}{2925}$$

$$\approx 0.41$$

Ex: Same company.
4 employees are randomly selected.
Probability that at least one drives?

UNORDERED

DRIVE	OTHERS
9	18

Total = 27

$$n(S) = C(27, 4) = 17550$$

Select 4

E = at least 1 drives

$n(E) =$ 1 drive and 3 others OR 2 OR 3 OR 4

E': no one drives

DRIVE	OTHERS
9	18

$$n(E') = C(18, 4) = 3060$$

$$Pr(E') = \frac{n(E')}{n(S)} = \frac{3060}{17550} \approx 0.17$$

$$Pr(E) = 1 - Pr(E')$$

Pr(E happens) = 1 − Pr(E doesn't happen)

$$Pr(E) = 1 - 0.17$$
$$\approx 0.83$$

Ex: A group of 10 people are randomly selected. Probability that at least 2 have last names starting with the same letter?

$$n(S) = \underbrace{26 \times 26 \times 26 \times \cdots \times 26}_{\text{Person 1} \quad 2 \quad \cdots \quad 10} = 26^{10}$$

E = at least 2 start with same letter

☹ n(E) = 2 the same or 3 the same or ... or 10 the same

E' = all start with different letters

$$n(E') = \underbrace{26}_{\text{Person 1}} \times \underbrace{25}_{2} \times \cdots \times \underbrace{17}_{\text{Person 10}}$$

$$= P(26, 10)$$

$$Pr(E') = \frac{n(E')}{n(S)} = \frac{P(26,10)}{26^{10}} \approx 0.14$$

$$\begin{aligned} Pr(E) &= 1 - Pr(E') \\ &\approx 1 - 0.14 \\ &\approx 0.86 \end{aligned}$$