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5.2 Binomial Experiments

repeated success-failure trials

Ex: Roll a die 8 times.
Probability that you get at most one 4?

$n = \#\text{ trials} = 8$

$p = P(\text{roll a } 4) = \frac{1}{6}$

$q = 1 - p = \frac{5}{6}$

$x = \#\text{ successes} = \#\text{ 4's rolled}$

$$P(x \le 1) = P(x=0) + P(x=1)$$

$$\boxed{{}_nC_x\, p^x q^{n-x}}$$

$$= \cancel{{}_8C_0}\left(\cancel{\tfrac{1}{6}}\right)^0\left(\tfrac{5}{6}\right)^8 + \cancel{{}_8C_1}\left(\tfrac{1}{6}\right)^1\left(\tfrac{5}{6}\right)^7$$

$$\approx 0.60$$

$${}_nC_x\, p^x q^{n-x}$$

Choose which trials are successes

$P(x \text{ successes})$

$P(n-x \text{ failures})$

Ex: A baseball player has a batting average of .450. He goes to bat 5 times.

a) Probability of more than 3 hits?

Binomial experiment

$n = 5$

$p = P(\text{hit}) = 0.45$

$q = 1 - p = 0.55$

$x = \#\text{ successes} = \#\text{ hits}$

$$P(x > 3) = P(x=4) + P(x=5)$$

$$\boxed{{}_nC_x\, p^x q^{n-x}}$$

$$= \cancel{{}_5C_4}(0.45)^4(0.55)^1 + \cancel{{}_5C_5}(0.45)^5\cancel{(0.55)^0}$$

$$\approx 0.13$$

b) Probability of at least 2 hits?

$$P(x \ge 2) = \cancel{P(x=2) + P(x=3) + \ldots}$$

$$= 1 - P(x=0) - P(x=1)$$

$$= 1 - \cancel{{}_5C_0}\cancel{(0.45)^0}(0.55)^5 - \cancel{{}_5C_1}(0.45)^1(0.55)^4$$

$$\boxed{{}_nC_x\, p^x q^{n-x}}$$

$$\approx 0.74$$

Expected value: theoretical $\boxed{\text{average}}$ if an experiment is repeated infinitely-many times

FACT

For a binomial experiment (repeated success-failure trials):

$$E(X) = np$$

Ex: 9% of manufactured items are defective. 30 items are randomly selected.

a) expected number of defective items selected?

Binomial experiment

$n = 30$
$p = P(\text{defective}) = 0.09$

$$E(X) = np$$
$$= 30(0.09)$$
$$= 2.7$$

b) P(3 defective items are selected)?

$n = 30 \quad p = 0.09$
$q = 1 - p = 0.91$
$x = \#\text{ successes} = \#\text{ defective items selected}$

$$P(x = 3) = \underbrace{\cancel{(30C3)}}_{4060}(0.09)^{3}(0.91)^{27}$$

$$\boxed{(nCx)\,p^{x}\,q^{n-x}}$$

$$\approx 0.23$$

S.2 #45

6% of manufactured items are defective. 10 items are randomly selected.

P(more than 2 defective items)?

Binomial

$n = 10$
$p = P(\text{defective}) = 0.06$
$q = 1 - p = 0.94$
$x = \#\text{ defective items}$

$$P(x > 2) = \cancel{P(x=3) + \ldots}$$
$$= 1 - P(x=0) - P(x=1) - P(x=2)$$
$$= 1 - \underbrace{\cancel{10C0}}_{1}\cancel{(0.06)^{0}}(0.94)^{10} - \underbrace{\cancel{10C1}}_{10}(0.06)^{1}(0.94)^{9} - \underbrace{\cancel{10C2}}_{45}(0.06)^{2}(0.94)^{8}$$

$$\boxed{(nCx)\,p^{x}\,q^{n-x}}$$

$$\approx 0.0188$$

Follow-Up: Find the probability distribution and draw a histogram.

(Same problem as above).

x = # defective items selected	Probability $= {}_{10}C_x (0.06)^x (0.94)^{10-x}$
0	${}_{10}C_0 (0.06)^0 (0.94)^{10} \approx 0.54$
1	${}_{10}C_1 (0.06)^1 (0.94)^9 \approx 0.34$
⋮	
10	${}_{10}C_{10} (0.06)^{10} (0.94)^0 \approx 0$