

5.2 Binomial Experiments

repeated success-failure trials

Ex: Roll a die 8 times.
Probability that you get at most one 4?

$$n = \# \text{ trials} = 8$$

$$p = P(\text{roll a } 4) = \frac{1}{6}$$

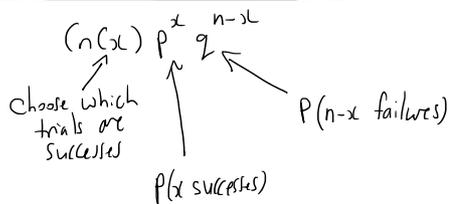
$$q = 1 - p = \frac{5}{6}$$

$$x = \# \text{ successes} = \# \text{ 4's rolled}$$

$$P(x \leq 1) = P(x=0) + P(x=1)$$

$$\boxed{(nCx) p^x q^{n-x}} = \cancel{8C0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^8 + \cancel{8C1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^7$$

$$\approx 0.60$$



Ex: A baseball player has a batting average of .450. He goes to bat 5 times.

a) Probability of more than 3 hits?

Binomial experiment

$$\begin{aligned} n &= 5 \\ p &= P(\text{hit}) = 0.45 \\ q &= 1 - p = 0.55 \\ x &= \# \text{ successes} = \# \text{ hits} \end{aligned}$$

$$P(x > 3) = P(x=4) + P(x=5)$$

$$\boxed{(nCx) p^x q^{n-x}} = \cancel{5C4} (0.45)^4 (0.55)^1 + \cancel{5C5} (0.45)^5 (0.55)^0$$

$$\approx 0.13$$

b) Probability of at least 2 hits?

$$P(x \geq 2) = \cancel{P(x=2) + P(x=3) + \dots}$$

$$= 1 - P(x=0) - P(x=1)$$

$$= 1 - \cancel{5C0} (0.45)^0 (0.55)^5 - \cancel{5C1} (0.45)^1 (0.55)^4$$

$$\boxed{(nCx) p^x q^{n-x}} \approx 0.74$$

Expected value: theoretical average if an experiment is repeated infinitely many times

FACT

For a binomial experiment (repeated success-failure trials):
 $E(x) = np$

Ex: 9% of manufactured items are defective.
30 items are randomly selected.

a) expected number of defective items selected?

Binomial experiment $n = 30$
 $p = P(\text{defective}) = 0.09$

$$\begin{aligned} E(x) &= np \\ &= 30(0.09) \\ &= 2.7 \end{aligned}$$

b) $P(3 \text{ defective items are selected})?$

$$n = 30 \quad p = 0.09$$

$$q = 1 - p = 0.91$$

$x = \# \text{ successes} = \# \text{ defective items selected}$

$$P(x = 3) = \frac{\binom{30}{3}}{4060} (0.09)^3 (0.91)^{27}$$

$$\boxed{(nCx) p^x q^{n-x}}$$

$$\approx 0.23$$

S.2 #45

6% of manufactured items are defective.
10 items are randomly selected.

$P(\text{more than 2 defective items})?$

Binomial

$$n = 10$$

$$p = P(\text{defective}) = 0.06$$

$$q = 1 - p = 0.94$$

$x = \# \text{ defective items}$

$$P(x > 2) = \cancel{P(x=3) + \dots}$$

$$= 1 - P(x=0) - P(x=1) - P(x=2)$$

$$= 1 - \cancel{\frac{10C0}{1}} (0.06)^0 (0.94)^{10} - \cancel{\frac{10C1}{10}} (0.06)^1 (0.94)^9 - \frac{10C2}{45} (0.06)^2 (0.94)^8$$

$$\boxed{(nCx) p^x q^{n-x}}$$

$$\approx 0.0188$$

Follow-up: Find the probability distribution
, and draw a histogram.

(Same problem as above).

$x = \# \text{ defective items selected}$	Probability = $\binom{10}{x} (0.06)^x (0.94)^{10-x}$
0	$\binom{10}{0} (0.06)^0 (0.94)^{10} \approx 0.54$
1	$\binom{10}{1} (0.06)^1 (0.94)^9 \approx 0.34$
\vdots	
10	$\binom{10}{10} (0.06)^{10} (0.94)^0 \approx 0$

