109Feb22

February 22, 2021 8:28 AM

5.1 Expected Value

Ex: Insure a tablet worth \$2000 against damage for 1 year. The premium is \$150.
P(damage) = 2.5%
Find your expected net gain.

X = net gain

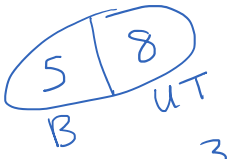
	X = net gain	P(X)
(damage)	2000 − 150 = 1850	0.025
(no damage)	−150	1 − 0.025 = 0.975

expected net gain

$$E(X) = x_1 p_1 + x_2 p_2$$
$$= 1850(0.025) - 150(0.975)$$
$$= -100$$

Interpretation: Expect to lose \$100 on the insurance policy. Protected against a large loss.

Ex: A class has 5 business students and 8 university-transfer students.
Choose 3 students randomly.
Let X = # of business students chosen.
Find the expected value of X.



Choose 3

X = # business	Description	# of Outcomes	$P(x)$
0	3 UT	$8C3 = 56$	$56/286$
1	1B and 2 UT	$5C1 \times 8C2 = 140$	$140/286$
2	2B and 1 UT	$5C2 \times 8C1 = 80$	$80/286$
3	3B	$5C3 = 10$	$10/286$
		Total = 286	

Notation: $C(8,3)$ or $8C3$

Expected value

$$E(X) = 0\left(\frac{56}{286}\right) + 1\left(\frac{140}{286}\right) + 2\left(\frac{80}{286}\right) + 3\left(\frac{10}{286}\right)$$

$$\approx 1.15$$

Interpretation: Expect to choose 1.15 business students, on average.

Def:

Fair game: expected net winnings = \$0

Ex: You pay \$2 to roll a die.
If you roll a 3 or 4, you win \$36.
Otherwise you must pay k more dollars.
Find k so that the game is fair.

X = net winnings

	X = net winnings	$P(x)$
(roll a 3,4)	$36 - 2 = 34$	$\frac{2}{6}$
(roll a 1,2,5,6)	$-2-k$	$\frac{4}{6}$

Fair game: Want $E(X) = 0$

$$34\left(\frac{2}{6}\right) + (-2-k)\left(\frac{4}{6}\right) = 0$$

Multiply by 6:

$$34(2) + (-2-k)(4) = 0$$

$$68 - 8 - 4k = 0$$

$$60 = 4k$$

$$k = 15$$

<u>Ex</u>: A card is drawn from a standard deck. If it's a Q or a diamond, you win \$1, otherwise you lose \$1. What is your expected winnings?

$X =$ winnings (\$)

$$\boxed{\begin{aligned} n(Q \text{ or diamonds}) &= n(Q) + n(\text{diamonds}) - n(Q \text{ and diamonds}) \\ &= 4 + 13 - 1 \\ &= 16 \end{aligned}}$$

	X	$P(X)$
(Q or diamond)	1	$\frac{16}{52}$
(otherwise)	-1	$1 - \frac{16}{52} = \frac{36}{52}$

expected winnings $E(X) = 1\left(\frac{16}{52}\right) + (-1)\frac{36}{52}$

$$\approx -0.38$$

Interpretation: Expect to lose \$0.38, on average.

Asst due Wed 2:30pm

Test Wed Mar 3 1:30-2:30

Covers Sections 4.3-4.6,
5.1-5.2, and
1.1-1.2