Camosun College
Department of Mathematics
Math 107 Practice Final
PART 1 of 2

Instructor: Susie Wieler

Name: Solutions.

Total Marks: [43]

Instructions:

- You have a total of 3 hours to write the exam.
- DO NOT start the exam until instructed to do so.
- Check that this booklet contains 10 questions (numbered 1-10).
- The only permissible calculator is the SHARP EL-531W, EL-531-X or EL-510R.
- Show all your work in the space provided. Marks will be deducted for incomplete work.
- NO DECIMALS are to be used in any answer, unless otherwise stated.
- This is PART 1 of your exam. Once you have finished it you may hand it in and have a break. If you do not wish to take a break, you may request PART 2 and keep PART 1 on your desk.

1. Solve $-2(x+3) < 8$. [3]

$x+3 > -4$

$x > -7$

2. Solve $2+\sqrt{4-2x} = x$. [3]

$\sqrt{4-2x} = x-2$

$4-2x = (x-2)^2$

$4-2x = x^2-4x+4$

$0 = x^2-2x$

$0 = x(x-2)$

$x=0$ or $x=2$

check for extraneous solutions:

$x=0$	$x=2$
$2+\sqrt{4-2\cdot 0} \stackrel{?}{=} 0$	$2+\sqrt{4-2\cdot 2} \stackrel{?}{=} 2$
$2+\sqrt{4} \stackrel{?}{=} 0$	$2+\sqrt{0} \stackrel{?}{=} 2$
$2+2 \neq 0$	$2+0 = 2$ ✓

so $\boxed{x=2}$

3. Find the equation of the line parallel to $2x + y = 2$ and containing the point $(4, 0)$. Leave your answer in slope-intercept form. [3]

$$y = -2x + 2$$

so $m = -2$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = -2(x - 4)$$
$$y = -2x + 8$$

4. Consider the equation $9x^2 - 18x + 4y^2 + 16y = 11$. Complete the square in both x and y and determine if the graph of this equation is a circle or an ellipse (you do not have to graph it). [3]

$$9x^2 - 18x + 4y^2 + 16y = 11$$
$$9(x^2 - 2x) + 4(y^2 + 4y) = 11$$
$$9(x^2 - 2x + 1) + 4(y^2 + 4y + 4) = 11 + 9 + 16$$
$$9(x-1)^2 + 4(y+2)^2 = 36$$
$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$$

This is an ellipse.

5. Let $f(x) = 4x + 3$. Find and simplify [3]

$$\frac{f(x+h) - f(x)}{h}.$$

$$\frac{4(x+h)+3-(4x+3)}{h}$$

$$= \frac{4x+4h+3-4x-3}{h}$$

$$= \frac{4h}{h}$$

$$= 4$$

6. Let $f(x) = x^2 + 2x$.

a) Graph $f(x)$. Label the y-intercept, x-intercepts, and vertex. [3]

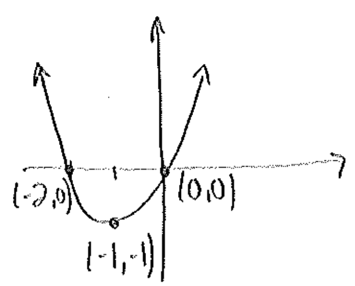
b) Determine the domain and range of $f(x)$. [2]

c) Determine where $f(x)$ is increasing and where it is decreasing. [2]

a) vertex: $\left(\frac{-b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

$\frac{-b}{2a} = -\frac{2}{2 \cdot 1} = -1$ $\quad f(-1) = (-1)^2 + 2(-1) = 1 - 2 = -1$

$x^2 + 2x = x(x+2)$ so the x-intercepts are $(0,0)$ and $(-2,0)$



b) domain $= \mathbb{R}$

range $= \{y \mid y \geq -1\} = [1, \infty)$

c) f is decreasing on $(-\infty, -1)$

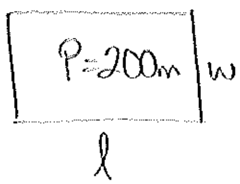
and increasing on $(-1, \infty)$

7. A farmer wishes to enclose a rectangular area with 200m of fencing.

a) Express the area A of the rectangle as a function of the width w of the rectangle. [1]

b) For what value of w is the area largest? [3]

c) What is the maximum area? [1]



a) $A = lw$

$P = 2l + 2w$

$200 = 2l + 2w$

$l = \frac{200 - 2w}{2} = 100 - w$

$A = (100 - w)w$

$A = 100w - w^2$

$A(w) = -w^2 + 100w$

b) to find the maximum of the quadratic $A(w)$, we need the vertex $\left(-\frac{b}{2a}, A\left(-\frac{b}{2a}\right)\right)$

$w = -\frac{b}{2a} = \frac{-100}{2(-1)} = 50m$

c) $A(50) = -50^2 + 100(50) = 2500\ m^2$

8. Let $f(x) = x^3 + 4x^2 + 4x$.

a) Factor $x^3 + 4x^2 + 4x$. [2]

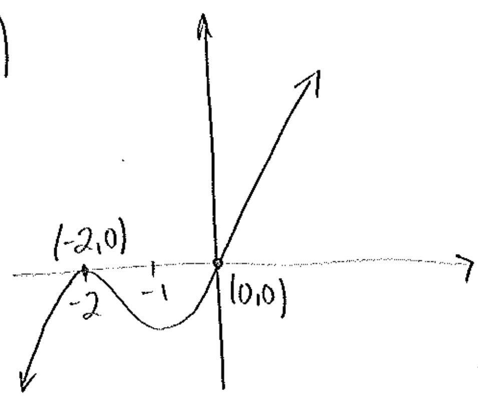
b) Graph $f(x)$. Label all the x-intercepts. [3]

c) Solve $f(x) > 0$. [1]

a) $x^3 + 4x^2 + 4x = x(x^2 + 4x + 4)$

$= x(x+2)^2$

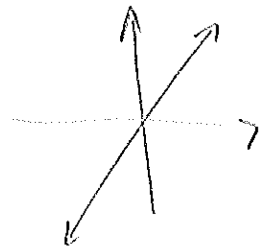
b)



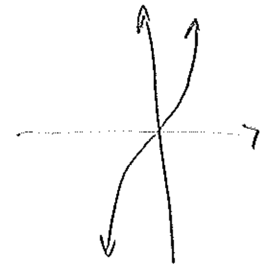
c) $x > 0$

9. Using an equation or a graph, provide an example of a one-to-one function. [2]

f(x) = x OR



OR



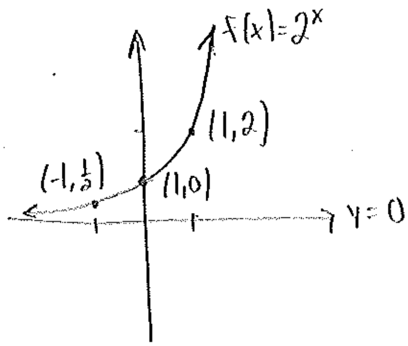
etc..

10. a) Graph $f(x) = 2^x$. Label at least three points on the graph. [2]

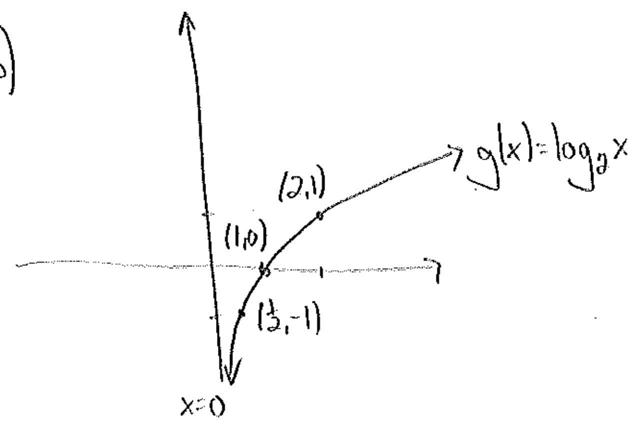
b) Graph $g(x) = \log_2 x$. Label at least three points on the graph. [2]

c) Graph $h(x) = \log_2(x + 1) - 2$. Label the asymptote, x-intercept, and y-intercept. Find the domain and range of $h(x)$. [4]

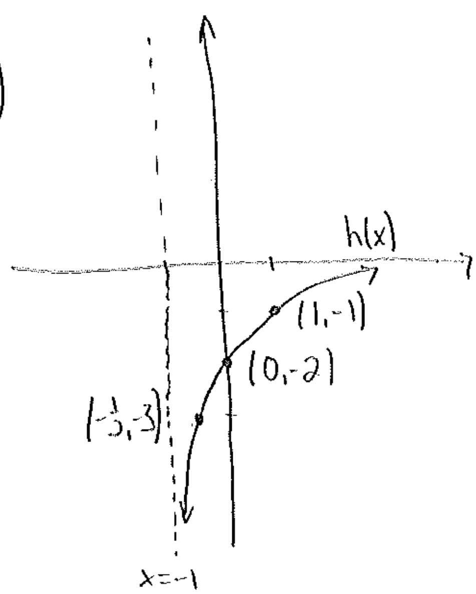
a)



b)



c)



domain: $x > -1$

range = $\mathbb{R}$

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PART 2 of 2

Instructor: Susie Wieler

Name: Solutions

Total Marks: [37]

Instructions:

- Check that this booklet contains 7 questions (numbered 11-18).
- The only permissible calculator is the SHARP EL-531W, EL-531-X or EL-510R.
- Show all your work in the space provided. Marks will be deducted for incomplete work.
- NO DECIMALS are to be used in any answer, unless otherwise stated.

11. Solve $\log_2(2x+1) = 3$. [3]

$2^3 = 2x+1$

$8 = 2x+1$

$2x = 7$

$x = \frac{7}{2}$

12. The population of a colony of mosquitoes obeys the law of uninhibited/exponential growth.

a) If there are 2000 mosquitoes initially and there are 2700 mosquitoes after 1 day, what is the size of the colony after 5 days? [3]

b) How long is it until there are 50,000 mosquitoes? [2]

$A = Pe^{rt}$

a) $P = 2000$, $A = 2700$ when $t = 1$

$2700 = 2000e^{r \cdot 1}$

$1.35 = e^r$

$r = \ln 1.35$

$A = 2000\, e^{(\ln 1.35)5} = 8968$

$e^{(\ln 1.35)t} = \left(e^{\ln 1.35}\right)^t$

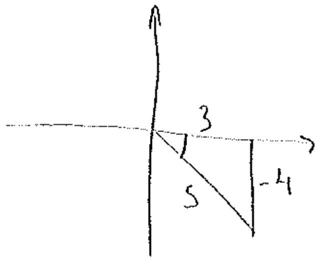
$= 1.35^t$

b) $50000 = 2000\, e^{(\ln 1.35)t}$

$25 = 1.35^t$

$t = \log_{1.35} 25 = \frac{\ln 25}{\ln 1.35} = 10.7$ days

13. Let $\cos\theta = \frac{3}{5}$ and let θ be in Quadrant IV. Find $\sin\theta$ and $\tan\theta$. [3]



$\sin\theta = -\frac{4}{5}$

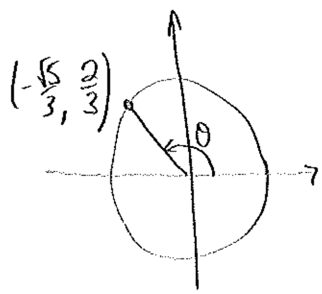
$\tan\theta = -\frac{4}{3}$

$\sqrt{5^2 - 3^2} = \sqrt{16} = 4$

14. The point $(-\frac{\sqrt{5}}{3}, \frac{2}{3})$ is on the unit circle and on the terminal side of an angle θ.

a) In which quadrant is θ? [1]

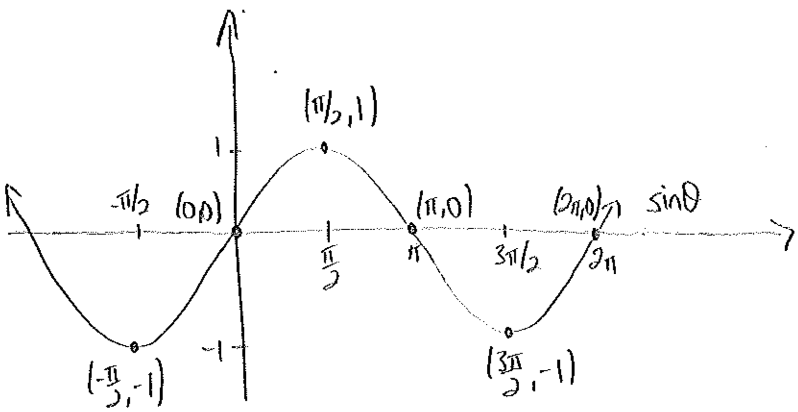
b) Find the exact value of $\cot\theta$. Remember to rationalize the denominator if necessary. [2]



a) Quadrant II

b) $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta} = \frac{-\frac{\sqrt{5}}{3}}{\frac{2}{3}} = -\frac{\sqrt{5}}{\cancel{3}} \cdot \frac{\cancel{3}}{2} = -\frac{\sqrt{5}}{2}$

15. Graph $f(\theta) = 3\sin(\pi\theta - 2) + 1$. Label at least six points on the graph. [5]



x-coordinates: mult. by $\frac{1}{\pi}$ then add $\frac{2}{\pi}$

y-coordinates: mult by 3 then add 1

$\left(-\frac{\pi}{2},-1\right) \longrightarrow \left(-\frac{1}{2}+\frac{2}{\pi},-2\right) \approx (0.1,-2)$

$(0,0) \longrightarrow \left(\frac{2}{\pi},1\right) \approx (0.6,1)$

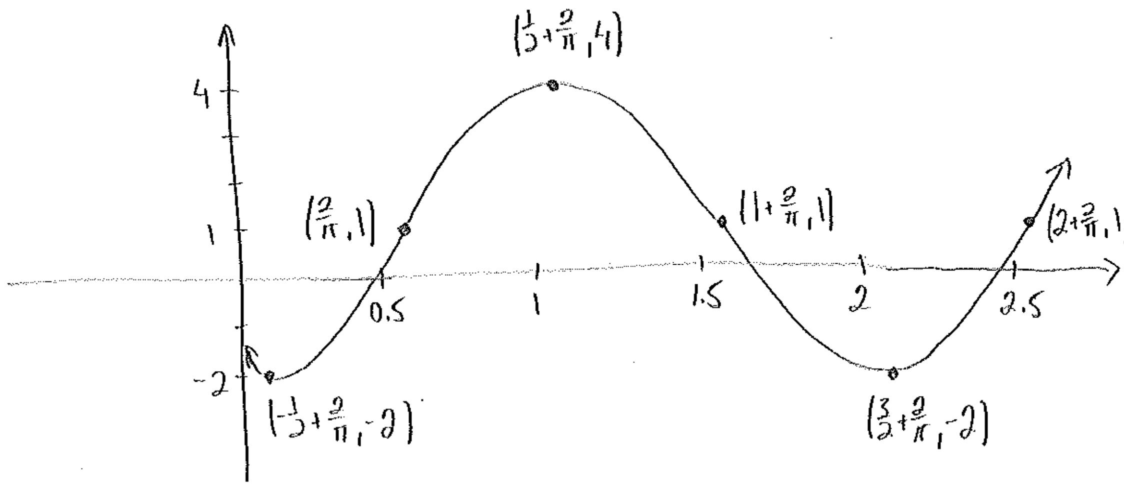
$\left(\frac{\pi}{2},1\right) \longrightarrow \left(\frac{1}{2}+\frac{2}{\pi},4\right) \approx (1.1,4)$

$(\pi,0) \longrightarrow \left(1+\frac{2}{\pi},1\right) \approx (1.6,1)$

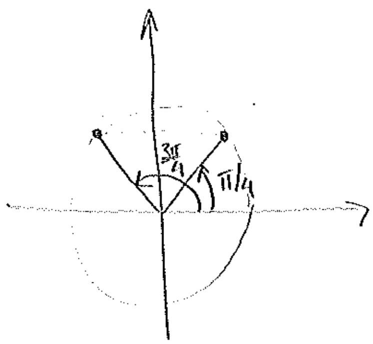
$\left(\frac{3\pi}{2},-1\right) \longrightarrow \left(\frac{3}{2}+\frac{2}{\pi},-2\right) \approx (2.1,-2)$

$(2\pi,0) \longrightarrow \left(2+\frac{2}{\pi},1\right) \approx (2.6,1)$

$\frac{2}{\pi} \approx 0.6$



16. Find the exact value of $\sin^{-1}(\sin \frac{3\pi}{4}) + \tan(\tan^{-1} \frac{3}{2})$. [4]



$\sin\frac{3\pi}{4} = \sin\frac{\pi}{4}$ and $\frac{\pi}{4}$ is in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

so $\sin^{-1}(\sin\frac{\pi}{4}) = \frac{\pi}{4}$

$$\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \tan\left(\tan^{-1}\frac{3}{2}\right)$$

$$= \sin^{-1}\left(\sin\frac{\pi}{4}\right) + \frac{3}{2}$$

$$= \frac{\pi}{4} + \frac{3}{2}$$

17. a) Derive the following double-angle formula: [3]

$$\cos(2\theta) = 1 - 2\sin^2\theta.$$

b) Use part a) to solve $\cos(2\theta) + 6\sin^2\theta = 4$ on the interval $0 \le \theta < 2\pi$. [4]

a) $\cos(2\theta) = \cos(\theta + \theta)$

$= \cos\theta\cos\theta - \sin\theta\sin\theta$

$= \cos^2\theta - \sin^2\theta$ and $\cos^2\theta + \sin^2\theta = 1$

$= (1 - \sin^2\theta) - \sin^2\theta$ so $\cos^2\theta = 1 - \sin^2\theta$

$= 1 - 2\sin^2\theta$

b) $\cos(2\theta) + 6\sin^2\theta = 4$

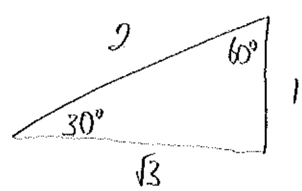
$1 - 2\sin^2\theta + 6\sin^2\theta = 4$

$4\sin^2\theta = 3$

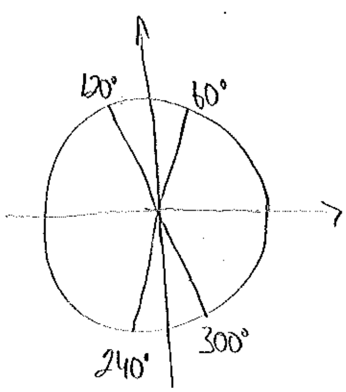
$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\sqrt{\frac{3}{4}}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$



$\sin 60° = \frac{\sqrt{3}}{2}$



$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$ or $\frac{5\pi}{3}$ radians

18. Consider the sequence

$$3, 6, 12, 24, \cdots.$$

a) Determine if this sequence is arithmetic, geometric, or neither. [2]

b) Find a formula for the n^{th} term of this sequence. [2]

c) Find the 94^{th} term of this sequence. [1]

d) Find the sum of the first 94 terms of this sequence. [2]

a) $\frac{6}{3} = \frac{12}{6} = \frac{24}{12} = 2$, so the sequence is geometric

b) $a_1 = 3$, $r = 2$ so

$$a_n = a_1 r^{n-1}$$

$$a_n = 3 \cdot 2^{n-1}$$

c) $a_{94} = 3 \cdot 2^{94-1} = 3 \cdot 2^{93} \simeq 2.97 \times 10^{28}$

d) $S_{94} = 3 \cdot \frac{1 - 2^{94}}{1-2} = -3(1 - 2^{94}) \simeq 5.94 \times 10^{28}$