

Solutions to Practice Test Questions

#32-47

32. $\ln e^{\sqrt{2}} = \log_e e^{\sqrt{2}} = \sqrt{2}$ (recall: $\log_a a^r = r$)

33. $2 \log_4 x = 1$ $x > 0$

$$\log_4 x = \frac{1}{2}$$

$$4^{1/2} = x$$

$$x = 2$$

34. $A = Pe^{rt}$

$$2P = Pe^{0.1t}$$

$$2 = e^{0.1t}$$

$$\ln 2 = 0.1t$$

$$t = \frac{\ln 2}{0.1} = 6.93 \text{ years}$$

35. a) $A = Pe^{rt}$

let's use $1800 = 1000e^{r(1)}$ to solve for r

$$1.8 = e^r$$

$$r = \ln 1.8$$

so $A = 1000e^{(\ln 1.8)(3)} = 5832$

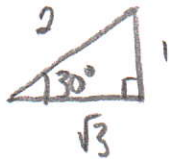
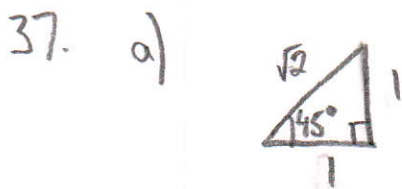
b) $10000 = 1000e^{(\ln 1.8)t}$

$$10 = e^{(\ln 1.8)t}$$

$$\ln 10 = (\ln 1.8)t$$

$$t = \frac{\ln 10}{\ln 1.8} = 3.9 \text{ days}$$

36. $\frac{5\pi}{6} \text{ rad} \left(\frac{180^\circ}{\pi \text{ rad}} \right) = 150^\circ$



$$2 \sin 45^\circ + 4 \cos 30^\circ$$

$$= 2 \left(\frac{1}{\sqrt{2}} \right) + 4 \left(\frac{\sqrt{3}}{2} \right) = \frac{2}{\sqrt{2}} + 2\sqrt{3} = \frac{2\sqrt{2}}{2} + 2\sqrt{3} = \sqrt{2} + 2\sqrt{3}$$

↑
rationalize

b) $\frac{\pi}{4} \text{ rad} = 45^\circ$, $\frac{\pi}{3} \text{ rad} = 60^\circ$

$$\sec \frac{\pi}{4} + 2 \csc \frac{\pi}{3} = \frac{1}{\cos 45^\circ} + 2 \left(\frac{1}{\sin 60^\circ} \right)$$

$$= \frac{\sqrt{2}}{1} + 2 \left(\frac{2}{\sqrt{3}} \right)$$

$$= \sqrt{2} + \frac{4}{\sqrt{3}} = \sqrt{2} + \frac{4\sqrt{3}}{3}$$

38. $\frac{\pi}{4} \text{ rad} = 45^\circ$

$$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\csc \frac{\pi}{4} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

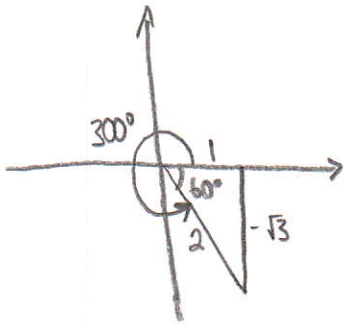
$$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sec \frac{\pi}{4} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\tan \frac{\pi}{4} = \frac{1}{1} = 1$$

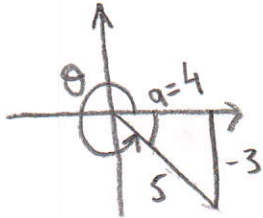
$$\cot \frac{\pi}{4} = \frac{1}{1} = 1$$

39.



$$\sec 300^\circ = \frac{1}{\cos 300^\circ} = \frac{2}{1} = 2$$

40.



$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$5^2 = (-3)^2 + a^2$$

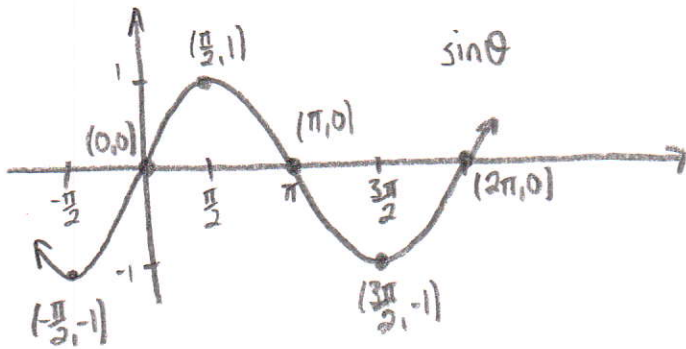
$$a^2 = 25 - 9 = 16$$

$$a = 4 \quad \text{since } a > 0$$

$$\cos \theta = \frac{4}{5}$$

$$\tan \theta = \frac{-3}{4}$$

41.



x-coords: $+\pi$

y-coords: $\times 2$

$$\left(-\frac{\pi}{2}, -1\right) \rightarrow \left(\frac{\pi}{2}, -2\right)$$

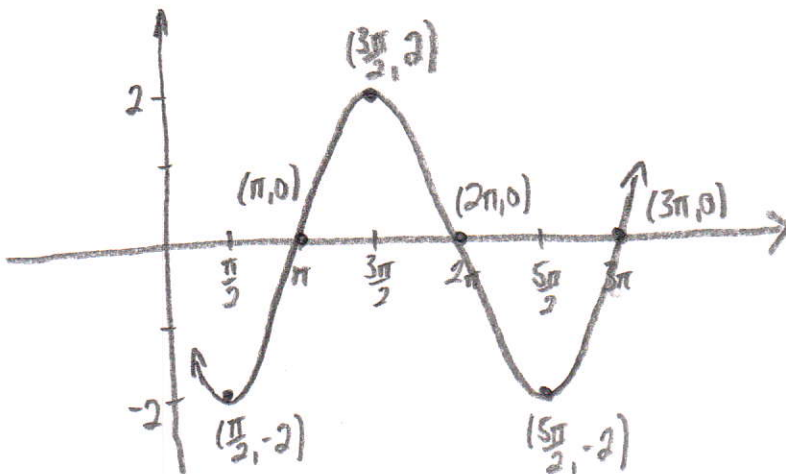
$$(0, 0) \rightarrow (\pi, 0)$$

$$\left(\frac{\pi}{2}, 1\right) \rightarrow \left(\frac{3\pi}{2}, 2\right)$$

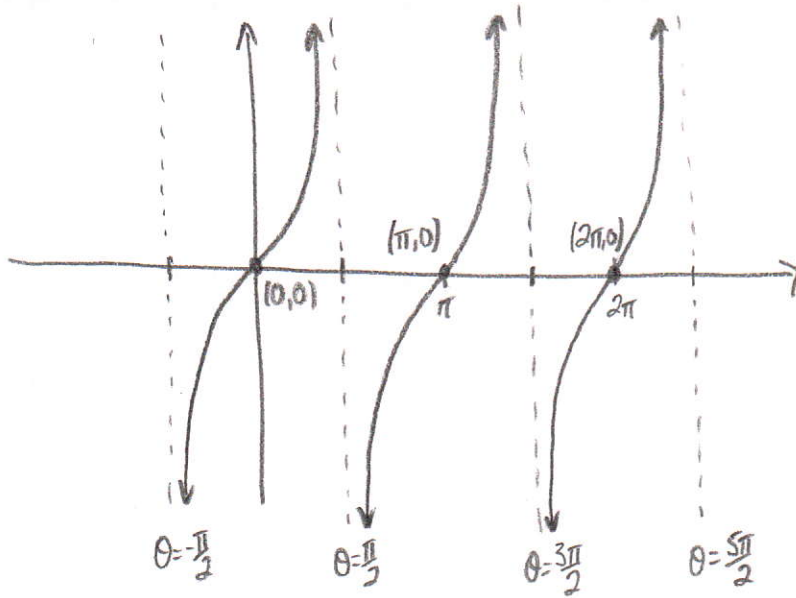
$$(\pi, 0) \rightarrow (2\pi, 0)$$

$$\left(\frac{3\pi}{2}, -1\right) \rightarrow \left(\frac{5\pi}{2}, -2\right)$$

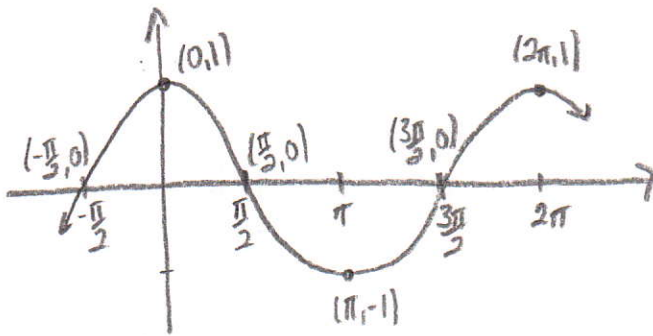
$$(2\pi, 0) \rightarrow (3\pi, 0)$$



42.



43. $f(\theta) = 3\cos(3\theta - \pi)$
 $= 3\cos\left[3\left(\theta - \frac{\pi}{3}\right)\right]$



x-coords: $\div 3$ then $+\frac{\pi}{3}$

y-coords: $\times 3$

$$\left(-\frac{\pi}{2}, 0\right) \longrightarrow \left(\frac{\pi}{6}, 0\right)$$

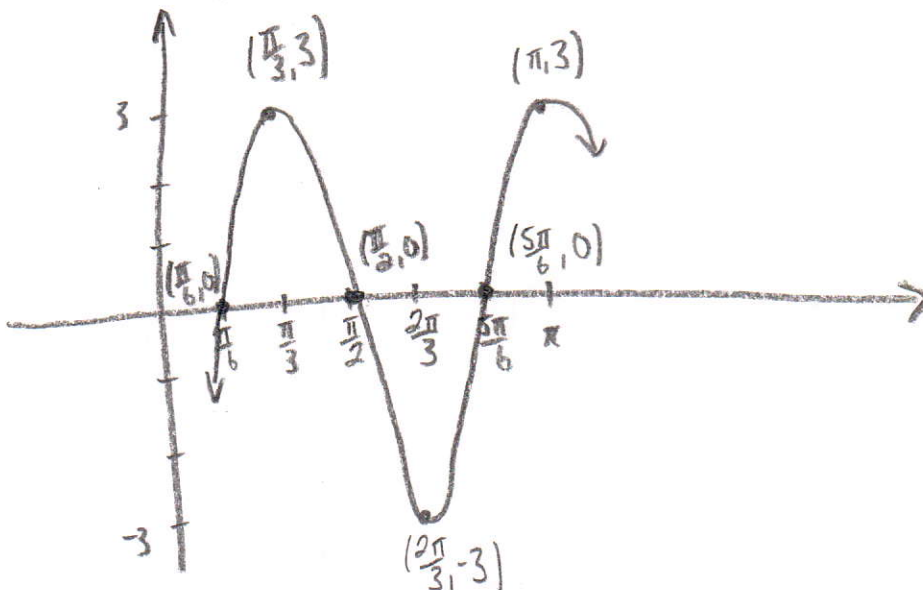
$$(0, 1) \longrightarrow \left(\frac{\pi}{3}, 3\right)$$

$$\left(\frac{\pi}{2}, 0\right) \longrightarrow \left(\frac{\pi}{2}, 0\right)$$

$$(\pi, -1) \longrightarrow \left(\frac{2\pi}{3}, -3\right)$$

$$\left(\frac{3\pi}{2}, 0\right) \longrightarrow \left(\frac{5\pi}{6}, 0\right)$$

$$(2\pi, 1) \longrightarrow (\pi, 3)$$



44. $\tan(\tan^{-1}4) = 4$

45. $\cos^2 \theta (1 + \tan^2 \theta) = 1$

recall: $\sec^2 \theta = 1 + \tan^2 \theta$

$$\begin{aligned} &\cos^2 \theta (1 + \tan^2 \theta) \\ &= \cos^2 \theta (\sec^2 \theta) \\ &= \cancel{\cos^2 \theta} \left(\frac{1}{\cancel{\cos^2 \theta}} \right) \\ &= 1 \quad \checkmark \end{aligned}$$

46. $\sin 15^\circ = \sin(45^\circ - 30^\circ)$

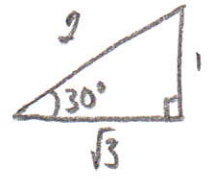
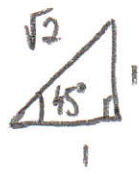
$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3} - 1}{2\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}} \right)$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$



47. $\cos^4 \theta - \sin^4 \theta = \cos(2\theta)$

$$\begin{aligned} &\cos^4 \theta - \sin^4 \theta \\ &= (\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta) \end{aligned}$$

$= (\cos^2 \theta - \sin^2 \theta)(1)$

$= \cos(2\theta) \quad \checkmark$