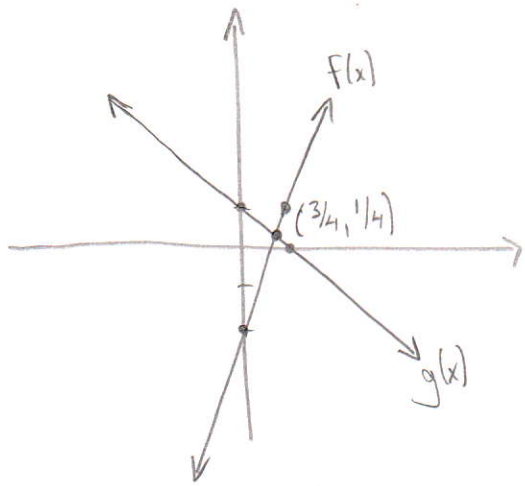
Solutions to Practice Test Questions

#20-31

20.



$f(x) = g(x)$

$3x - 2 = -x + 1$

$4x = 3$

$x = \frac{3}{4}$

$f\left(\frac{3}{4}\right) = 3\left(\frac{3}{4}\right) - 2 = \frac{9}{4} - \frac{8}{4} = \frac{1}{4}$

$\left(\frac{3}{4}, \frac{1}{4}\right)$

21. $f(x) = -2x^2 + 12x$

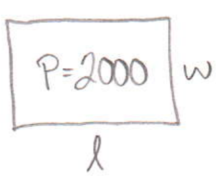
vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

$-\frac{b}{2a} = -\frac{12}{2(-2)} = 3$

$f(3) = -2(3)^2 + 12(3) = -18 + 36 = 18$

$\boxed{(3, 18)}$

22.



a) $A = wl$, but we want A as a function of w only

$P = 2w + 2l$

$2000 = 2w + 2l$

$1000 = w + l$

$l = 1000 - w$, substitute into $A = wl$

$A = w(1000 - w)$

$\boxed{A = 1000w - w^2}$

b) $A = -w^2 + 1000w$ looks like 

to find the maximum, we need the vertex

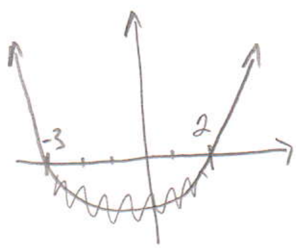
$\frac{-b}{2a} = \frac{-1000}{2(-1)} = 500$ this is the width that gives the max. area

At $w = 500$, $A = -(500)^2 + 1000(500) = \boxed{250{,}000 \text{ m}^2}$

23. $x^2 < -x + 6$

$x^2 + x - 6 < 0$

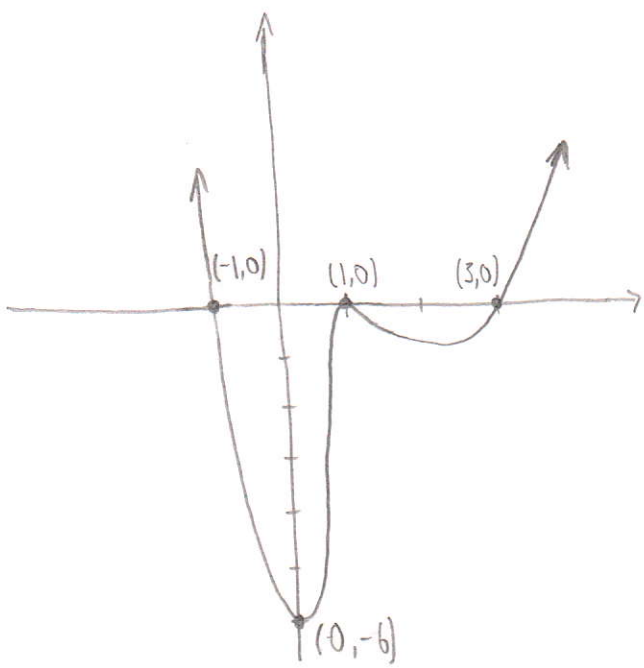
$(x+3)(x-2) < 0$



$\boxed{-3 < x < 2}$

Note: this can also be solved using the number line method or by solving the 2 cases $[x+3<0$ and $x-2>0]$ or $[x+3>0$ and $x-2<0$

24. $f(x) = 2(x-1)^2(x-3)(x+1) = 2x^4 + \ldots$



y-intercept:

$f(0) = 2(0-1)^2(0-3)(0+1)$

$= -6$

25. $f(x) = \dfrac{3x+5}{x-6}$

vertical asymptote: $x=6$

horizontal asymptote: $y = \frac{3}{1} = 3$

x-intercept: $f(x)=0$

$$\frac{3x+5}{x-6} = 0$$

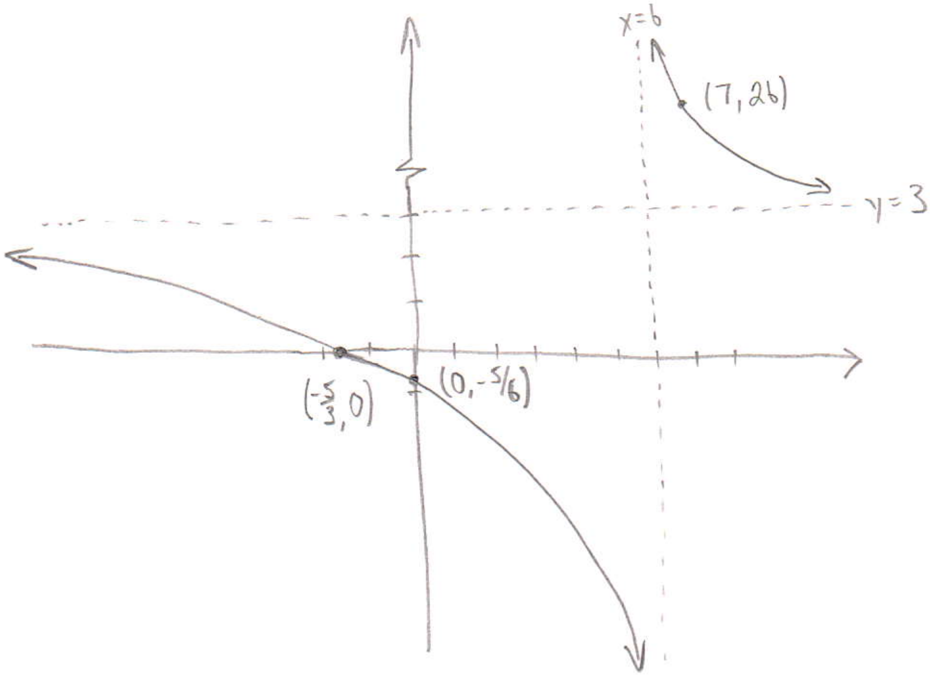
$$3x+5=0$$

$$3x=-5$$

$$x=-\frac{5}{3}$$

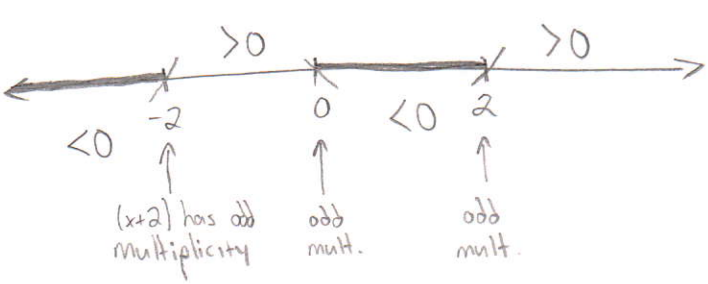
y-intercept:

$$f(0) = \frac{3\cdot 0+5}{0-6} = -\frac{5}{6}$$



$$f(7) = \frac{3\cdot 7+5}{7-6} = 26$$

26. $\dfrac{(x-2)(x+2)}{x} \le 0$ note that $x \neq 0$



$\boxed{x \le -2 \quad \text{or} \quad 0 < x \le 2}$

start with the left interval $(-\infty, -2$

choose a number in this interval

$x=-3$ $\quad \dfrac{(x-2)(x+2)}{x} = \dfrac{(-3-2)(-3+2)}{-3} = -\dfrac{5}{3}$

since $-\frac{5}{3} < 0$, $\dfrac{(x-2)(x+2)}{x} < 0$

on the interval $(-\infty, -2)$

27. $f(x) = 6x^4 - x^3 + 2x^2 - 4x - 3$

potential rational zeros:

factors of -3: $\pm 1, \pm 3$
factors of 6: $\pm 1, \pm 2, \pm 3, \pm 6$

so zeros could be $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 3, \pm \frac{3}{2}$

check for actual zeros:

$x = 1$ $\quad f(1) = 6(1)^4 - 1^3 + 2(1)^2 - 4(1) - 3 = 0$

since $f(1) = 0$, $\boxed{x = 1}$ is a zero of $f(x)$.

(note: this means $x-1$ is a factor of $f(x)$.)

28. a) $(f \circ g)(1) = f(g(1)) = f\left(\frac{4 \cdot 1}{1-2}\right) = f(-4) = \frac{1}{-4} = \boxed{-\frac{1}{4}}$

b) domain of $g(x) = \{x \neq 2\}$

domain of $f(x) = \{x \neq 0\}$. We need $g(x)$ to be in the domain of $f(x)$, i.e. we need $g(x) \neq 0$

$$\frac{4x}{x-2} \neq 0$$

$$4x \neq 0$$

$$x \neq 0$$

So the domain of $f \circ g$ is $\{x \neq 2, x \neq 0\}$

29. a) $f(x) = \frac{3x}{x-2}$

$$x = \frac{3y}{y-2}$$

$$x(y-2) = 3y$$

$$xy - 2x = 3y$$

$$xy - 3y = 2x$$

$$(x-3)y = 2x$$

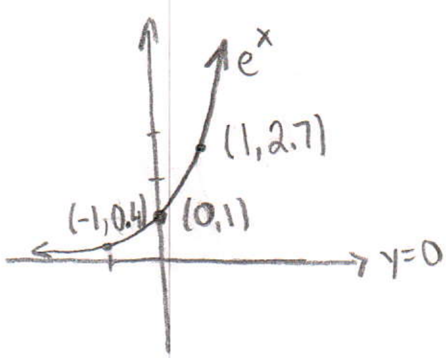
$$y = \frac{2x}{x-3}$$

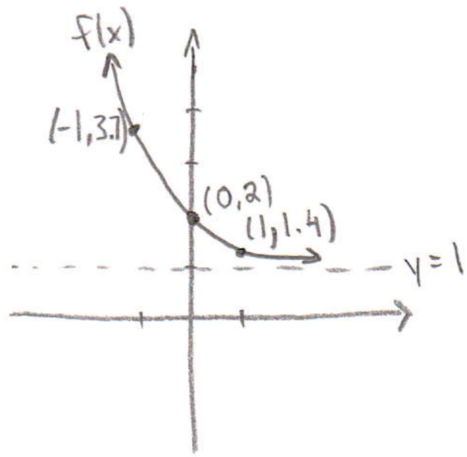
$$\boxed{f^{-1}(x) = \frac{2x}{x-3}}$$

b) domain of f = $\{x \neq 2\}$

range of f = domain of f^{-1} = $\{x \neq 3\}$

30.





$e \approx 2.7$

$\frac{1}{e} \approx 0.4$

31. range of f = domain of f^{-1} = $\{x - 2 > 0\}$

$= \boxed{\{x > 2\}}$