Solutions to Practice Test Questions

1. $x^2 + 5x + 6 = (x+2)(x+3)$

2. $$\begin{array}{r|rrrr} 1 & 1 & 2 & 0 & 1 \\ & & 1 & 3 & 3 \\ \hline & 1 & 3 & 3 & \boxed{4} \end{array}$$

quotient $= x^2 + 3x + 3$

remainder $= 4$

3. $$\frac{4}{x-1} + \frac{x}{x+2} = \frac{4(x+2)}{(x-1)(x+2)} + \frac{x(x-1)}{(x+2)(x-1)} = \frac{4x+8 + x^2 - x}{(x+2)(x-1)}$$

$$= \frac{x^2+3x+8}{(x+2)(x-1)}$$

4. $$\frac{(x^2y)^{1/3}(xy^3)^{2/3}}{(xy)^2} = \frac{x^{2/3}y^{1/3}x^{2/3}\cancel{y^2}}{x^2\cancel{y^2}} = \frac{x^{4/3}y^{1/3}}{x^2} = x^{-2/3}y^{1/3} = \frac{y^{1/3}}{x^{2/3}}$$

5. $$\frac{2x}{x^2-4} = \frac{4}{x^2-4} - \frac{3}{x+2}$$

$x \neq \pm 2$

$$(x-2)(x+2)\left(\frac{2x}{(x-2)(x+2)}\right) = \left(\frac{4}{(x-2)(x+2)} - \frac{3}{x+2}\right)(x-2)(x+2)$$

$$2x = 4 - 3(x-2)$$
$$2x = 4 - 3x + 6$$
$$5x = 10$$
$$x = 2$$

but $x \neq 2$ so there is $\boxed{\text{no solution}}$

6. $$x^2 + x = 4$$
$$x^2 + x + \tfrac{1}{4} = 4 + \tfrac{1}{4}$$
$$\left(x + \tfrac{1}{2}\right)^2 = \frac{17}{4}$$
$$x + \tfrac{1}{2} = \pm\sqrt{\frac{17}{4}}$$

$$x = -\frac{1}{2} \pm \frac{\sqrt{17}}{2} = \frac{-1 \pm \sqrt{17}}{2}$$

7. $2x^{2/3} - 5x^{1/3} - 3 = 0$

let $u = x^{1/3}$

then $u^2 = x^{2/3}$

$2u^2 - 5u - 3 = 0$

$2u^2 - 6u + u - 3 = 0$

$2u(u-3) + (u-3) = 0$

$(2u+1)(u-3) = 0$

$2u+1 = 0$ or $u - 3 = 0$

$u = -\frac{1}{2}$ $\qquad$ $u = 3$

$x^{1/3} = -\frac{1}{2}$ $\qquad$ $x^{1/3} = 3$

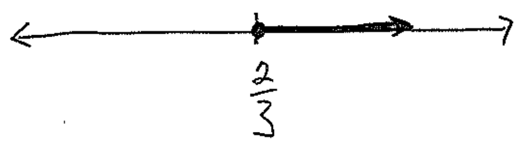
$\boxed{x = -\frac{1}{8}}$ or $\boxed{x = 27}$

8. $4 - 3x \le 2$

$-3x \le -2$

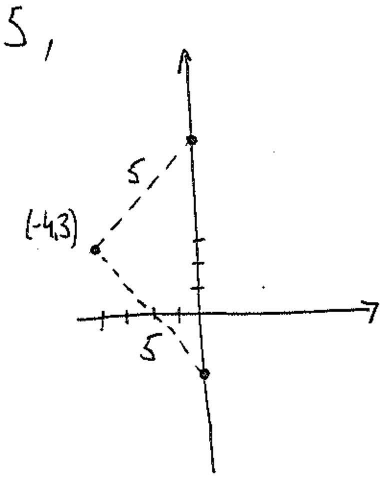
$x \ge \frac{2}{3}$

$[\frac{2}{3}, \infty)$



9. distance between $(0,y)$ and $(-4,3)$ is 5,

we want to solve for y



$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$5 = \sqrt{(-4-0)^2 + (3-y)^2}$

$5 = \sqrt{16 + 9 - 6y + y^2}$

$5 = \sqrt{y^2 - 6y + 25}$

$25 = y^2 - 6y + 25$

$0 = y^2 - 6y$

$0 = y(y-6)$

so $y = 0$ or $y = 6$

$\boxed{(0,0) \text{ and } (0,6)}$

10. x-intercepts: $y=0$, solve for x

$0^2 = x+9$

$x=-9$

$\boxed{(-9,0)}$

y-intercepts: $x=0$, solve for y

$y^2 = 0+9$

$y^2 = 9$

$y = \pm 3$

$\boxed{(0,3) \text{ and } (0,-3)}$

symmetry w.r.t. the x-axis:

replace y by $-y$

$(-y)^2 = x+9$

$y^2 = x+9$ ✓ so it is symmetric w.r.t. the x-axis

symmetry w.r.t. the y-axis:

replace x by $-x$

$y^2 = -x+9$ X so it is not symmetric w.r.t. the y-axis

symmetry w.r.t. the origin:

replace x by $-x$ and y by $-y$

$(-y)^2 = -x+9$

$y^2 = -x+9$ X so it is not symmetric w.r.t. the origin

11. $x - 2y = 3$

$-2y = -x+3$

$y = \frac{1}{2}x - \frac{3}{2}$

So $m = \frac{1}{2}$

$y = mx+b$, solve for b using $(1,2)$

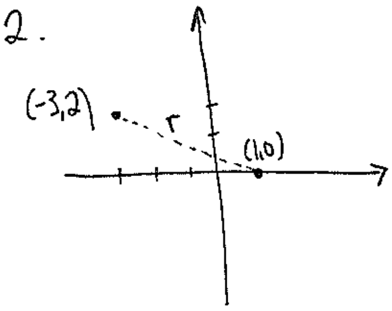
$2 = \frac{1}{2}(1) + b$

$2 = \frac{1}{2} + b$

$\frac{3}{2} = b$

$\boxed{y = \frac{1}{2}x + \frac{3}{2}}$

12.



the radius of the circle is the distance between (1,0) and (-3,2)

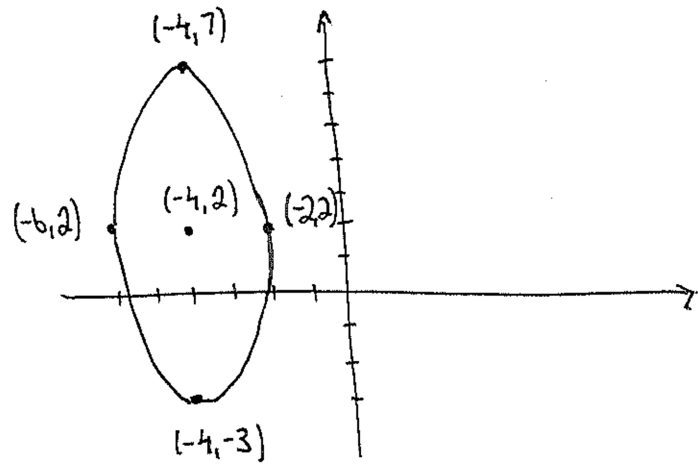
$$r = \sqrt{(-3-1)^2 + (2-0)^2}$$

$$= \sqrt{16 + 4}$$

$$= \sqrt{20}$$

$$\boxed{(x-1)^2 + y^2 = 20}$$

13.



14.

$$\frac{f(x+h) - f(x)}{h} = \frac{[3(x+h)^2 + 1] - (3x^2+1)}{h}$$

$$= \frac{[3(x^2 + 2xh + h^2) + 1] - 3x^2 - 1}{h}$$

$$= \frac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{1} - \cancel{3x^2} - \cancel{1}}{h}$$

$$= \frac{h(6x + 3h)}{h}$$

$$= 6x + 3h$$

15. $f(x) = \dfrac{x^2-1}{x^2-x-2} = \dfrac{x^2-1}{(x-2)(x+1)}$ so $x \neq 2, -1$

x-intercept(s): set $f(x)=0$ and solve for x

$0 = \dfrac{x^2-1}{x^2-x-2}$

$0 = x^2-1$ $\boxed{(1,0)}$

$x^2 = 1$

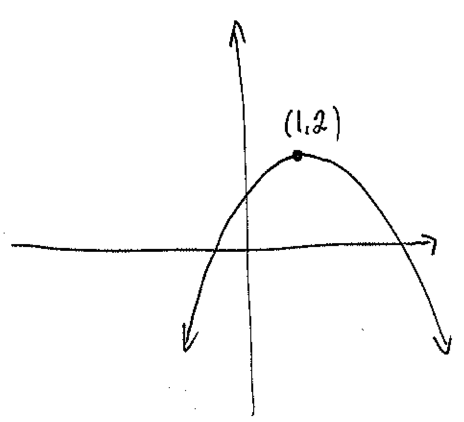
$x = \pm 1$, but $x \neq -1$

y-intercept: $(0, f(0))$

$f(0) = \dfrac{0^2-1}{0^2-0-2} = \dfrac{-1}{-2} = \dfrac{1}{2}$

$\boxed{(0, \tfrac{1}{2})}$

16. many possible solutions

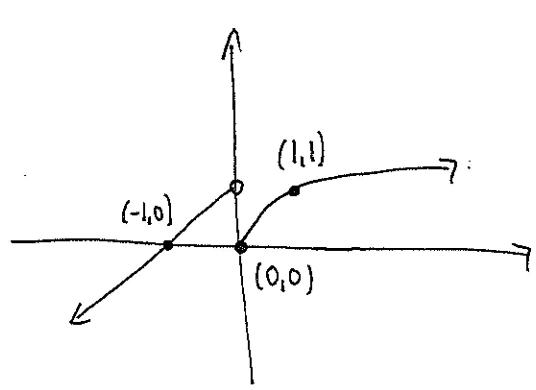


17. a) $f(-1) = -1+1 = 0$

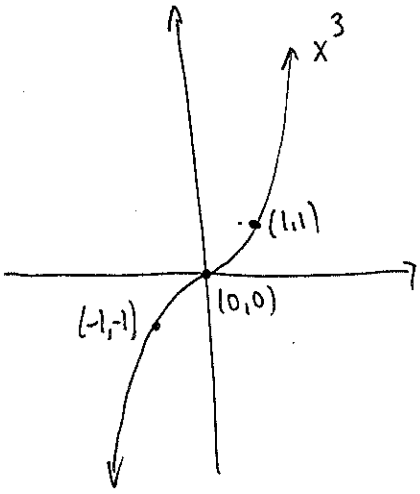
$f(0) = \sqrt{0} = 0$

$f(1) = \sqrt{1} = 1$

b)



18.



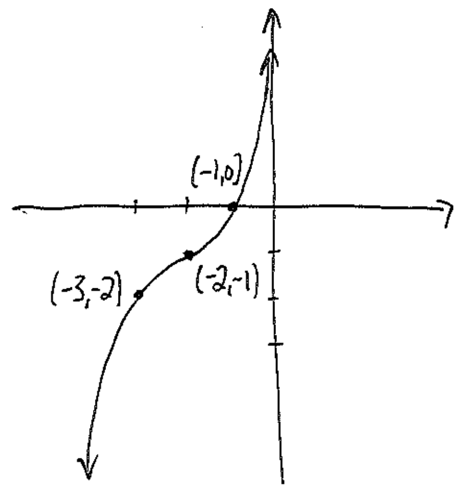
x-coordinates -2

y-coordinates -1

$(-1,-1) \rightarrow (-3,-2)$

$(0,0) \rightarrow (-2,-1)$

$(1,1) \rightarrow (-1,0)$



19.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$(x_1, y_1) = (x, f(x))$$

$$(x_2, y_2) = (0, 0)$$

$$d = \sqrt{(0 - x)^2 + (0 - f(x))^2}$$

$$d = \sqrt{x^2 + (x^2 - 1)^2}$$

$$d = \sqrt{x^2 + x^4 - 2x^2 + 1}$$

$$d(x) = \sqrt{x^4 - x^2 + 1}$$