

RS. n^{th} Roots ; Rational Exponents

p.1

$$\underline{\text{Ex:}} \quad \sqrt{9} = \sqrt{3^2} = 3$$

$${}^4\sqrt{16} = {}^4\sqrt{2^4} = 2$$

$${}^3\sqrt{-8} = {}^3\sqrt{(-2)^3} = -2$$

$${}^3\sqrt{\frac{-1}{27}} = {}^3\sqrt{\left(\frac{-1}{3}\right)^3} = \frac{-1}{3}$$

${}^n\sqrt{a}$ is undefined when n is even and $a < 0$

Ex: $\sqrt{-1}$ and ${}^4\sqrt{-1}$ are undefined

$${}^3\sqrt{-1} = {}^3\sqrt{(-1)^3} = -1$$

Caution: ${}^n\sqrt{a^n} = |a|$ when n is even and $a < 0$

Ex: ${}^4\sqrt{(-2)^4} = {}^4\sqrt{16} = 2$ (not -2)

$${}^3\sqrt{(-7)^3} = -7$$

$${}^n\sqrt{ab} = {}^n\sqrt{a} \cdot {}^n\sqrt{b} \quad {}^n\sqrt{\frac{a}{b}} = \frac{{}^n\sqrt{a}}{{}^n\sqrt{b}} \quad {}^n\sqrt{a^m} = ({}^n\sqrt{a})^m$$

Ex: $\sqrt{128} = \sqrt{64} \sqrt{2} = 8\sqrt{2}$

$\sqrt[3]{24} = \sqrt[3]{8} \cdot \sqrt[3]{3} = 2 \cdot \sqrt[3]{3}$

$\sqrt[4]{16^3} = (4\sqrt[4]{16})^3 = 2^3 = 8$

$\sqrt{2} \sqrt{8} = \sqrt{16} = 4$

$\sqrt[4]{\frac{81x^5}{16}} = \frac{\sqrt[4]{81} \sqrt[4]{x^4} \sqrt[4]{x}}{\sqrt[4]{16}} = \frac{3|x| \cdot \sqrt[4]{x}}{2}$

Like Radicals

Ex: $-3\sqrt{28} + 4\sqrt{7}$
 $= -3\sqrt{4\sqrt{7}} + 4\sqrt{7}$
 $= -6\sqrt{7} + 4\sqrt{7}$
 $= -2\sqrt{7}$

Rationalizing the Denominator

No roots in denominator

Ex: a) $\frac{7}{4\sqrt{3}} = \frac{7\sqrt{3}}{4\sqrt{3}\sqrt{3}} = \frac{7\sqrt{3}}{12}$

b) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1}{(\sqrt{5}+\sqrt{2})} \cdot \frac{(\sqrt{5}-\sqrt{2})}{(\sqrt{5}-\sqrt{2})} = \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$

$$c) \frac{1}{\sqrt[3]{2}} = \frac{1}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{\sqrt[3]{4}}{2}$$

p.3

$$a^{1/n} = \sqrt[n]{a}$$

Ex: $16^{1/2} = \sqrt{16} = 2$

$$27^{1/3} = 3$$

$$(-8)^{1/3} = -2$$

$$a^{m/n} = \sqrt[n]{a^m} \text{ or } (\sqrt[n]{a})^m$$

Ex: $(-32)^{3/5} = (\sqrt[5]{-32^3})^3 = (-2)^3 = -8$

Negative exponents

Ex: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$$8^{-4/3} = \frac{1}{8^{4/3}} = \frac{1}{(\sqrt[3]{8})^4} = \frac{1}{16}$$

Ex: Let $x > 0$ and $y > 0$. Simplify and express with positive exponents.

a) $\left(\frac{16xy^{1/5}}{x^{1/3}y} \right)^{1/2} = (16x^{2/3}y^{-4/5})^{1/2}$

$$\begin{aligned} x^{1-\frac{1}{3}} &= x^{2/3} \\ x^{1/3} &= x^{2/3} \end{aligned}$$

$$= 16^{1/2} x^{1/3} y^{-2/5}$$

$$= \frac{4x^{1/3}}{y^{2/5}}$$

$$(x^{2/3})^{1/2} = x^{\frac{2}{3} \cdot \frac{1}{2}} = x^{1/3} \quad \text{p.4}$$

$$b) \left(\frac{3x^{1/2}}{y^{3/2}} \right)^{-2} = \left(\frac{y^{3/2}}{3x^{1/2}} \right)^2 = \frac{y^3}{9x}$$

Ex: Write as a single quotient with positive exponents

$$(x^2+2)^{1/2} + 4x^2(x^2+2)^{-1/2}$$

$$= \frac{(x^2+2)}{(x^2+2)^{1/2}} + \frac{4x^2}{(x^2+2)^{1/2}} \quad \leftarrow \text{Mult. by } \frac{(x^2+2)^{1/2}}{(x^2+2)^{1/2}}$$

$$= \frac{5x^2+2}{(x^2+2)^{1/2}}$$

Ex: Factor and simplify

$$\frac{4}{5}x^{1/2}(3x+1) + 2x^{3/2} \quad \leftarrow \frac{10}{5}x^{3/2}$$

$$= \frac{2}{5}x^{1/2} [2(3x+1) + 5x]$$

$$= \frac{2x^{1/2}(11x+2)}{5}$$