6.1 Composite Functions

$$\boxed{(f \circ g)(x) = f(g(x))}$$

"f composed with g"

<u>Ex</u>: Let $f(x) = x^2 - 5$ and $g(x) = x^3 + 1$. Find:

a) $(f \circ g)(x)$

$= f(g(x))$

$= f(x^3+1)$

$= (x^3+1)^2 - 5$

b) $(g \circ f)(x)$

$= g(f(x))$

$= g(x^2-5)$

$= (x^2-5)^3 + 1$

c) $(f \circ f)(x)$

$= f(f(x))$

$= f(x^2-5)$

$= (x^2-5)^2 - 5$

d) $(f \circ g)(1)$

$= f(g(1))$ $\boxed{g(1) = 2}$

$= f(2)$

$= 2^2 - 5$

$= -1$

e) $(g \circ f)(1)$

$= g(f(1))$ $\boxed{f(1) = -4}$

$= g(-4)$

$= (-4)^3 + 1$

$= -63$

f) $(g \circ g)(1)$

$= g(g(1))$ $\boxed{g(1) = 2}$

$= g(2)$

$= 2^3 + 1$

$= 9$

$$(f \circ g)(x) = f(g(x))$$

$$x \xrightarrow[\text{apply } g]{} g(x) \xrightarrow[\text{apply } f]{} f(g(x))$$

Domain of $f \circ g$:

x must be in the domain of g

<u>and</u>

$g(x)$ " f

<u>Ex</u>: Let $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x-1}$

Domain of $f \circ g$?

$(f \circ g)(x) = f(g(x))$

x in domain of g : $x \neq 1$

<u>and</u>

$g(x) = \frac{1}{x-1}$ in domain of f : $\frac{1}{x-1} \geq 0$

$x - 1 \geq 0$

$x \geq 1$

$\boxed{x \neq 1 \text{ and } x \geq 1}$

$\boxed{x > 1}$

Ex: Let $f(x) = \frac{1}{x-3}$ and $g(x) = \frac{5}{x+1}$

a) Find $(g \circ f)(x)$

$$= g(f(x))$$

$$= g\left(\frac{1}{x-3}\right)$$

$$= \frac{5}{\frac{1}{x-3} + 1}$$

Multiply by $\frac{x-3}{x-3}$

$$= \frac{5(x-3)}{1 + x - 3}$$

$$= \frac{5(x-3)}{x-2}$$

b) Domain of $g \circ f$

$$(g \circ f)(x) = g(f(x))$$

x in domain of f: $x \neq 3$

and

$f(x) = \frac{1}{x-3}$ in domain of g: $\frac{1}{x-3} \neq -1$

$$1 \neq -(x-3)$$

$$1 \neq -x+3$$

$$-2 \neq -x$$

$$x \neq 2$$

$$\boxed{x \neq 2, 3}$$

<u>Ex</u>: Find f and g so that

a) $(f \circ g)(x) = \sqrt{x^4 - 8}$

$g(x) = x^4 - 8$

$f(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x)) = \sqrt{x^4 - 8}$ ✓

b) $(f \circ g)(x) = (x^3 + 1)^8$

$g(x) = x^3 + 1$

$f(x) = x^8$

$(f \circ g)(x) = f(g(x)) = (x^3 + 1)^8$ ✓

c) $(f \circ g)(x) = \dfrac{1}{x^2 + 1} + 3$

$g(x) = x^2 + 1$

$f(x) = \dfrac{1}{x} + 3$

$(f \circ g)(x) = f(g(x)) = \dfrac{1}{x^2 + 1} + 3$ ✓