

## 13.2 Arithmetic Sequences

P.1

A sequence is arithmetic if the difference between consecutive terms is constant.

$d$ : Common difference

Ex:  $\underbrace{7, 10, \dots}_{+3}, \underbrace{13, \dots}_{+3}, \underbrace{16, \dots}_{+3}$  is arithmetic  
1st term  $a_1 = 7$

$d$  = common difference

$$= a_n - a_{n-1} \quad (\text{any term} - \text{previous term}) \\ = 3$$

Ex: Is the sequence arithmetic? If so, find  $a_1$  and  $d$

a)  $\{a_n\} = \{2^n\}$

$$2, 4, 8, 16, \dots$$

Not arithmetic

b)  $\{a_n\} = \{2 - 7n\}$

$$-5, -12, -19, \dots$$

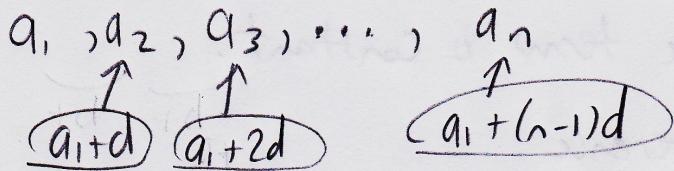
Arithmetic ✓

$$\begin{aligned} a_1 &= -5 \\ d &= a_n - a_{n-1} \\ &= -12 - (-5) \\ &= -7 \end{aligned}$$

$n^{\text{th}}$  term of an arithmetic sequence

P.2

$$a_n = a_1 + (n-1)d$$



Ex: Find the 30<sup>th</sup> term of 3, 8, 13, 18, ...

Arithmetic    a<sub>1</sub> = 3    d = 5

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned} a_{30} &= a_1 + 29d \\ &= 148 \end{aligned}$$

Ex: An arithmetic sequence has a<sub>5</sub> = -3  
and a<sub>10</sub> = -13. Find a formula for a<sub>n</sub>.

→ Find a<sub>1</sub> and d

$$a_5 = -3$$

$$a_{10} = -13$$

$$a_1 + 4d = -3 \quad ①$$

$$a_1 + 9d = -13 \quad ②$$

$$② \quad a_1 + 9d = -13$$

$$- ① \quad a_1 + 4d = -3$$

$$5d = -10$$

$$d = -2$$

+ either equation

$$① \quad a_1 + 4(-2) = -3$$

$$a_1 = 5$$

P.9

$$a_n = a_1 + (n-1)d$$

$$a_n = s + (n-1)(-2)$$

or  $a_n = 7 - 2n$

P.3

$$[a_1 + a_n] \frac{n}{2} = 12$$

$$[5 + 5] \frac{12}{2} = 12$$

$$10 \cdot 6 = 60$$

Sum of the first  $n$  terms of an arithmetic sequence

$$S_n = \frac{n}{2}[a_1 + a_n]$$

or  $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

Ex: Find the sum of the first 20 terms of  
6, 7.5, 9, ...

Arithmetic  $a_1 = 6$   $d = 1.5$   $n = 20$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$S_{20} = \frac{20}{2}[2(6) + 19(1.5)] \\ = 405$$

Ex: Find the sum  $2 + 5 + \dots + 200$

Arithmetic  $a_1 = 2$   $d = 3$   $a_n = 200$

$$n = ?$$

$$a_n = 200$$

$$a_1 + (n-1)d = 200$$

$$2 + (n-1)(3) = 200$$

$$n-1 = 66$$

$$\boxed{n=67}$$

$$\text{Ex 8.9 } S_n = \frac{n}{2} [a_1 + a_n]$$

p. 4

$$S_{67} = \frac{67}{2} [2 + 200]$$

$$= 6767$$

Recursive definition of a sequence : define  $a_n$  in terms of previous terms

Recursive definition of an arithmetic sequence :

$$a_n = a_{n-1} + d \text{ and indicate } a_1 \text{ term}$$

Ex: Give a recursive definition of a)  $3, 5, 7, \dots$

$$a_n = a_{n-1} + 2 \quad a_1 = 3$$

$n \geq 2$

b)  $-7, -9, -11, \dots$

$$a_n = a_{n-1} - 2 \quad a_1 = -7$$

$n \geq 2$

$$\text{constant } + \text{term } + \text{multiple of } n \text{ term}$$

$$\text{constant } = b, \text{ term } = d, \text{ multiple of } n \text{ term } = nd$$

$$c = 7$$

$$cd = 7d$$

$$cd = b(n-1) + d$$

$$cd = (2)(n-1) + 5$$

$$cd = nd$$

$$f_2 = n$$