Math 251 X01 & X02
Assignment

Total: 25 marks

Name: _____

Due: At the beginning of class, Monday April 8th
Covers Sections: 4.3-4.4 and 5.1-5.3

INSTRUCTIONS
* Submit your work on this paper.
* If you are away on the duedate then submit via the D2L Dropbox.
* You may discuss with others but your write-up must be your own work.
* Show all your work for full marks.

1. [3 marks] $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal basis for $\mathbb{R}^3$, where:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} \text{ and } \mathbf{v}_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}.$$

Write $\mathbf{w} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ as a linear combination of the basis vectors.

2. [7 marks] Find an orthogonal basis for $\text{span}\left(\begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 0 \\ 2 \end{bmatrix}\right)$.

3. [5 marks] Let $W = \text{span}\left(\begin{bmatrix} 1 \\ 0 \\ -8 \\ 121 \\ -43 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 9 \\ -140 \\ 22 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -23 \\ 347 \\ -126 \end{bmatrix} \right).$

Find a basis for $W^{\perp}$.

4. [4 marks] $A = \begin{bmatrix} 5 & 2 & 0 \\ -1 & 3 & 1 \\ 0 & 1 & 5 \end{bmatrix}$ has characteristic equation $(\lambda-4)^2(\lambda-5) = 0$.

a) Find the algebraic multiplicity of $\lambda = 4$.

b) Find the geometric multiplicity of $\lambda = 4$.

c) Is A diagonalizable? Explain.

5. [6 marks] The matrix A has eigenvalue 2 corresponding to the eigenvector $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and eigenvalue 3 corresponding to the eigenvector $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

Find the top-left entry of A^n.